The Inner Planets and the Keplerian Revolution

by

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How are the Ptolemaic, Copernican, and Keplerian theories of the inner planets related to one another structurally, and how do they compare in accuracy? The following analysis, making use of Bryant Tuckerman's *Planetary, Lunar, and Solar Positions, A.D. 2. to A.D. 1649*,[1] as well as modern orbital elements extrapolated backwards in time, seeks to throw some new light on these questions. It will become apparent that the pre-Keplerian theories, while mirroring in a rough way certain general features of the motions of Venus and Mercury, were too inaccurate to justify the claim that they "manage to account satisfactorily for all naked-eye observations."[2] At the same time, the analysis will elucidate the empirical basis for the Ptolemaic devices—equant point and bisection of the eccentricity—that figure in the derivation of Kepler's theory, and will indicate how the Ptolemaic and Copernican theories yielded clues supporting Kepler's major hypothesis—the dynamic action of the Sun.

Introductory remarks: the problem of the inner planets as it confronted Kepler.

As is well known, the route leading to Kepler's "new astronomy" involved primarily and unavoidably the study of the motions of Mars; for this planet alone had the requisite combination of properties—large enough eccentricity, nearness to the Earth, observability of heliocentric longitudes in all parts of the orbit. As Kepler puts it, "It was altogether necessary that we should either come to know the secrets of astronomy from the motions of Mars, or else remain perpetually ignorant of them."[3] But precisely because of the central premiss of the Keplerian program—its

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affirmation of a single Sun-focused dynamics as determining the motions of all the planets—it was essential to its success that the characteristic features found in the motions of Mars be shown to be applicable to the other circumsolar planets as well. The application in the case of the remaining superior planets, Jupiter and Saturn, was relatively straightforward. The changes required for the inner planets were rather more abrupt and striking.

From an early stage in Kepler's speculations, the inner planets had confronted him with a puzzle, because their motions as postulated by Ptolemy and Copernicus failed to accord with the heliocentric vision from which he set out. According to this vision, first formulated in print in the *Mysterium Cosmographicum* of 1596, the motive virtue whereby the planets were moved had its source in the Sun, and decreased in strength with increasing distance from this source. Each planet was linked dynamically to the Sun; but between the various circumsolar planets no dynamic, causal linkage was imagined or assumed—each had its own separate theory, relating it to the Sun alone. In support of this hypothesis, Kepler showed in Chapter 20 of the *Mysterium* that of any two circumsolar planets the one farther from the Sun moved more slowly in its path; and in Chapter 22 that each of the superior planets as presented in Copernican theory moved more rapidly or more slowly according as it was closer to or farther from the Sun. The Copernican theories of Venus and Mercury, however, violated this pattern by incorporating motions that keep time with the Earth's annual motion. Kepler admits this difficulty, along with another having to do with the Earth's motion, in the concluding paragraph of Chapter 22:

> But ... nothing is in every respect happy. For in Venus and Mercury this slowness and quickness is accommodated not to the distance of the planet from the Sun, but solely to the motion of the Earth. And if anyone assumes a different condition of motion for these planets from that found in the superior planets, what cause will he then propose in the case of the annual motion of the Earth? For neither according to Ptolemy nor according to Copernicus does this motion require an equant [a center about which the motion is angularly uniform, but which is not identical with the center of the planet's circular path, and which as a consequence implies that the planet's motion along its path is non-uniform]. Therefore, let this controversial question remain in suspense before the astronomer as judge.[4]

The key thought required for a resolution of the problem first occurred to Kepler in late spring of 1601, when he was away from his books, having journeyed from Prague to Graz on business connected with his wife's inheritance; so we learn from a letter of June, 1601, to Magini in Bologna.[5] Already by this time Kepler had derived from the Tychonic observations of Mars a demonstration that the Earth increases and decreases its orbital speed according as it approaches or recedes from the Sun, and thus obeys the norm followed by the superior planets in the Copernican reformulation of Ptolemaic theory. This conclusion can be stated in a different way: contrary to the implication of Ptolemaic astronomy (when translated into heliostatic form), and contrary to the explicit assumption of Copernicus,the center of the Earth's orbit was not identical with the so-called equant point—the point about which the planet was assumed to move with uniform angular speed; rather, the center of the orbit was only about half as far from the Sun as the equant point. This "bisection of the eccentricity", as it was called, now suggested itself to Kepler in Graz as an explanation for the disturbing motions in the Ptolemaic and Copernican theories of Venus and Mercury—the motions so incongruously commensurable with the Earth's motion: "these supposed inequalities of the inferior planets are nothing else than the parallax resulting from the motion of approach and recession of the Earth, hitherto insufficiently known ..."[6]

Actually, as Kepler points out many years later in Book VI of his *Epitome astronomiae Copernicanae* (1621), the situation is more complicated: "both eccentricities, that of the great orb [the orbit of the Earth] as well as that of the eccentric of the planet [Venus or Mercury], were confounded in one in the astronomy of the ancients."[7] But, as also becomes evident from Kepler's discussion in the *Epitome*, the bisection of the eccentricity in the Ptolemaic theory of Venus is from Kepler's point of view a reflection of the bisection of the eccentricity of the Sun or Earth, the eccentricity of Venus being comparatively small. In fact, Venus theory may well have been the locus of Ptolemy's first discovery or introduction of the bisection of the eccentricity, for in the case of Venus, Ptolemy presents the bisected eccentricity as emerging directly from and indeed imposed by the appearances. Ptolemy can then have transferred this device to the superior planets, where the observational data do not of themselves suggest and do not provide so direct a confirmation of the bisection. It is probably a result of the lack of direct confirmation in the case of the superior planets that later astronomers could come to look upon the bisection as (in Dreyer's phrase) a "perfectly arbitrary assumption".[8]

Under the Copernican transformation, the bisection as it occurs in the Ptolemaic theory of Venus is replaced by the incongruous annual oscillation of the orbit of Venus over which Kepler puzzled. In the case of the superior planets, the bisection remains though in somewhat disguised form; deprived of its disguise by Maestlin, it becomes for Kepler a principal clue to his celestial physics.[9] On the basis of observational data, Tycho denies the quantitative exactitude of the bisection in the case of Mars, but Kepler's analysis leads him to conclude that, when Tycho's point has been rightly assessed, the eccentricity will nevertheless be found to be exactly bisected; and it is from this conclusion that he proceeds to the abandonment of the equant principle, to the attempt to derive the planetary motions from an explicit celestial dynamics, and thus to "the renovation of all astronomy." A plausible reconstruction of the prehistory of the Keplerian ideas must thus accord a significant role to the Ptolemaic theory of Venus.

The case of Mercury is markedly different from that of Venus. Mercury's eccentricity is huge; the center of Mercury's orbit is over four and a half times farther from the Sun than the center of the Earth's orbit. It is one consequence of this difference that, whereas Ptolemy's theories of Venus and the superior planets are formally alike, involving epicycle and deferent and equant point with bisected eccentricity, and differing only in their numerical parameters, the Ptolemaic theory of Mercury is more complicated, resembling in this the Ptolemaic theory of the Moon: the center of the deferent is moved on a hypocycle and the epicycle is brought closest to the Earth not at one position 180° from the apogee of the deferent, as with the other planets, but at two positions 120° from the apogee on either side. In the Ptolemaic System, the extra complication in the theory of Mercury has a certain plausibility; as Ptolemy puts it in his *Planetary Hypotheses*, "The spheres nearest to the air [namely, those of the Moon and Mercury] move with many kinds of motion and resemble the nature of the element adjacent to them"—resemble, that is, the changeable air.[10] With the Copernican transformation, however, the complications cease to have even this vague cosmological suitability for support.

We have seen that Kepler from the beginning of his speculations looked with suspicion on the incorporation into the theories of the inner planets of motions with the periodicity of the Earth. When in the late spring of 1601 he realized that part of the complication in Mercury's case could arise from the failure of Ptolemy and Copernicus to bisect the eccentricity of the Earth (or Sun), he was still laboring under the conviction that the planetary orbits were perfectly circular, and as a consequence he was exceedingly doubtful concerning the truth of the double perigee of Mercury.[11] With his later discovery of the oval shape of the orbit, he comes to believe the mystery solved; and so in the *Epitome* he asserts that it was the elliptical shape of the orbit that forced Ptolemy to establish two perigees for Mercury.[12] At the same time, he describes the observations used in the construction of the Ptolemaic theory as rough, and implies that they were tailored a bit to yield a symmetrical theory. [13] Although (as will be shown) Mercury does not in fact have two perigees, we shall see that Kepler's second conclusion is plausible: the double perigee, though wrong, can be an exaggerated reflection of the ellipticity and inclination of Mercury's orbit.

It is thus evident that, from the time of the *Mysterium Cosmographicum*, Kepler saw the Ptolemaic and Copernican theories of Venus and Mercury as confronting him with puzzles demanding resolution. There are indications that he was wrestling with these problems even during the early years of his "war against Mars", and hoping to resolve them quickly.[14] His ultimate success in the case of Mercury turned out to depend on the prior resolution of the problem of Mars, and it then constituted a significant victory for his Martian paradigm, influential in the after years as leading to more general acceptance of the Keplerian ellipse and the Keplerian tables. In the case of Venus, an empirically satisfactory theory would have required less: circular orbits and equant points, for both the Earth and Venus, would have sufficed. Yet even had Kepler here retained rather than discarded these devices, his accomplishment would nevertheless have constituted a new and revolutionary decipherment of the appearances.

Venus

We first review the general phenomena that Ptolemy's theory of Venus is designed to "save", and then turn to an examination of the specific observations from which he derives the numerical parameters of his theory.

On the average Venus completes its longitudinal revolution, or circuit eastward about the zodiac, in the same time as the Sun does. Another way of stating this same fact is to say that the angular distance or elongation of Venus from the Sun never exceeds a fixed limit, about 48°; thus as the Sun proceeds eastward along the ecliptic at its nearly constant rate, averaging a little less than 1° per day, Venus overtakes it and then falls behind it again, but is never more than 48° away from it. This irregularity in the motion of Venus may be referred to as the *heliacal anomaly*, because it relates to the Sun. Ptolemy's mode of accounting for it is to assume that Venus moves round an epicycle, while the center of the epicycle moves eastward on a deferent circle with the mean angular speed of the Sun. Positions of Venus on the epicycle when it is at maximum elongation are such that the line of sight from the Earth to Venus is tangent to the epicycle. The direction of motion of Venus on the epicycle is determined by the observation that it takes Venus more time (indeed about three times more time) to pass from a greatest western elongation (when it is a morning star) to a greatest eastern elongation (when it is an evening star), than to pass in the reverse direction, from a greatest eastern to a greatest western elongation. This fact shows that Venus as it would be observed from the ecliptic's north pole is moving counterclockwise on its epicycle. Ptolemy finds the average time for completion of one cycle of heliacal anomaly (from greatest eastern elongation, say, back to greatest eastern elongation again) to be very nearly 8/5 of a year or 584 days.

Besides the heliacal anomaly, there is another irregularity, the *zodiacal anomaly*. This may be characterized in general terms as consisting in the fact that the greatest western and eastern elongations of Venus have different values in different parts of the zodiac. Ptolemy measures the elongations from the "Mean Sun" rather than from the "True and Apparent Sun", the Mean Sun being a point imagined as moving uniformly along the ecliptic with the average rate of the Sun. The position of the Mean Sun coincides with that of the True Sun when the latter is at apogee or perigee, and differs from the latter at other times; the maximum difference, 2°23′ according to Ptolemy, occurs when the Sun is about midway between apogee and perigee. In terms of elongations from the Mean Sun, the zodiacal anomaly as Ptolemy believes it to be has the following features: (a) For any given position of the Mean Sun in the zodiac, there is one definite recurrent greatest western elongation, and one definite recurrent greatest eastern elongation, these two elongations not being in general equal to one another. (b) At two positions of the Mean Sun, 180° apart, the two elongations are equal. (c) The sum of the greatest western and the greatest eastern elongations is a maximum for one and only one position of the Mean Sun in the zodiac, and on either side of that point becomes progressively less until it is a minimum at the opposite part of the zodiac. The two positions for the maximum and minimum sums are the same as the two positions mentioned in (b) above, where western and eastern elongations are equal. It is a peculiar feature of the anomaly that the largest maximum elongation does not occur where the largest sum of elongations occurs, but rather at two symmetrically placed positions on either side. A similar statement can be made with regard to the smallest maximum elongation.

Given the Ptolemaic account of the heliacal anomaly, the variation in maximum elongations from one point to another of the zodiac, or zodiacal anomaly, suggests that the Earth is not at the center of the deferent circle but rather off-center; for on this assumption the distance between the Earth and the epicycle will vary and as a consequence the angle between the tangents drawn from the Earth to the epicycle will vary, as required by phenomenon (c) above. The first requisite for quantifying the theory is then to determine the direction of the line of apsides, or line through the Earth and the center of the deferent. Ptolemy does this by seeking and finding two positions of the Mean Sun such that Venus's greatest eastern elongation from one of them is equal to her greatest western elongation from the other; on the assumption of symmetry, the line of apsides should pass midway between the two positions. Ptolemy's result from two pairs of observations is that the line of apsides points toward 25° Scorpion and 25° Bull. But how accurate is this result?

Using orbital elements computed by linear extrapolation for A.D. 140 from modern values,[15] one finds the perigee of the Earth's orbit at 71°0′ and its eccentricity 0.01749; the perigee of Venus's orbit at 105°23.4′, and its eccentricity 0.007698, or 0.005682 in terms of the Earth's mean solar distance taken as unity, where Venus's mean solar distance is 0.72333. The inclination of Venus's orbit to the ecliptic, about 3°22.3′ in A.D. 140, affects the heliocentric longitudes of Venus by only 3′, and the elongations by 6′ at maximum, and will be neglected in the calculations that follow.

The geometrical relations involved in determining where Ptolemy ought to have found the perigee and apogee of Venus are shown, with eccentricities exaggerated, in Figure I: S is the Sun, F the center of the Earth's orbit, and D the center of Venus's orbit; SF = 0.01749, SD = 0.005682, and angle FSD = 34°23.4′. Solution of the triangle shows angle SFD to be 13°42.4′; hence the line FD connecting the centers of the orbits points to 57°17.6′. The departure of the orbits from circularity is exceedingly slight—particularly so in the case of Venus. As a consequence the point P, on FD extended, is the point of the Earth's orbit that most nearly ap-

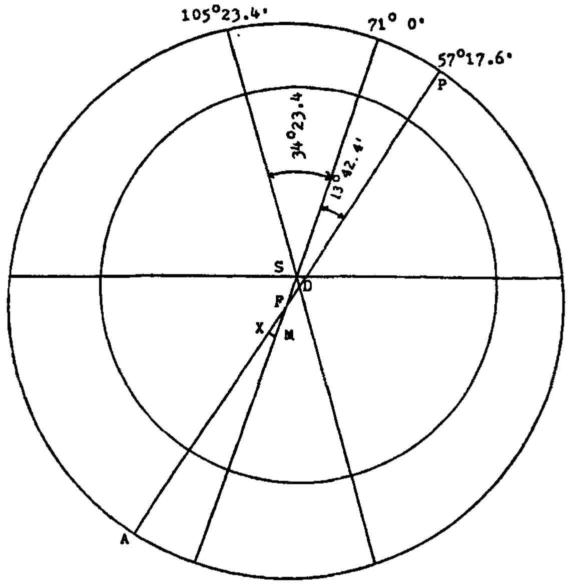

Fig. I

proaches the orbit of Venus. Hence the greatest eastern and western elongations of Venus as observed from P yield a larger sum than do the greatest elongations observed from any other point of the Earth's orbit.

In another respect, however, the point P differs from the perigee described in Ptolemaic theory: the greatest eastern and western elongations of Venus from the Mean Sun, as observed from P, are not equal. With an error not exceeding about 16″ of arc, the point M on the line of apsides of the Earth's orbit, located so that SM is twice SF, may be used as a Ptolemaic-style equant point; to this approximation it represents the Mean Sun. But M is not on the line of centers FD; its distance from that line is such as to subtend at A or P an angle of about 15′. It follows that the greatest western elongation observed from P should exceed the greatest eastern elongation by about twice 15′, or half a degree, while the greatest eastern elongation observed from A should exceed the greatest western elongation by about the same amount. The fact that Ptolemy's theory implies almost exactly equal elongations for this position of the Earth must prepare us for considerable errors in his observations; thus we shall not be able to endorse the conclusion of a recent study, according to which the observations of maximum elongations used by Ptolemy have a precision of $\pm$ 10.6′.[16] At the same time, it is apparent that the assumption of symmetry in Ptolemy's theory is wrong, and could be shown to be so on the basis of determinations of maximum elongations accurate to within 10′ of arc. Were we to select a point on the Earth's orbit near P from which Venus's maximum elongations from the Mean Sun would appear equal, it would have to be on the line through D and M, and the longitude of the position would be found to be about 65°5.5′.

The two pairs of observations used by Ptolemy in *Almagest* X, 1 to determine the position of perigee and apogee are compared in the following table with the corresponding numbers derived from Tuckerman's ephemeris.

Following Manitius and Czwalina,[17] we take the year of the second observation to be 4 Antonine rather than the 14 Antonine given in Heiberg's text of the Almagest; in fact, the position Ptolemy gives for the

Table I

Observation	Ptolemy	Tuckerman	Δ (P − T)
1. 21−22 Pharmouthi, 16 Hadrian = 8 March 132 (evening)			
Longitude of Venus........	31°30′	33° 0′	−1°30′
Longitude of Mean Sun.....	344°15′	345°22′	−1° 7′
Elongation................	47°15′	47°37′	−0°22′
2. 11−12 Thoth, 4 Antonine = 30 July 140 (morning)			
Longitude of Venus........	78°30′	80°34′	−2° 4′
Longitude of Mean Sun.....	125°45′	126°46′	−1° 1′
Elongation................	47°15′	46°12′	+1° 3′
3. 21−22 Athyr, 12 Hadrian = 12 October 127 (morning)			
Longitude of Venus........	150°20′	151°45′	−1°25′
Longitude of Mean Sun.....	197°52′	199° 3′	−1°11′
Elongation................	47°32′	47°18′	+0°14′
4. 9−10 Mechir, 21 Hadrian = December 136 (evening)			
Longitude of Venus........	319°36′	320° 3′	−0°27′
Longitude of Mean Sun.....	272° 4′	273°10′	−1° 6′
Elongation................	47°32′	46°53′	+0°39′

Mean Sun at the time of the second observation is derivable by way of his tables of the Sun's regular movement for the morning of 11 – 12 Thoth, 4 Antonine, but is incompatible with the morning of 11 – 12 Thoth, 14 Antonine. The combination of Mean Sun positions and planetary positions compared with the Tuckerman ephemeris suffices to fix the dates with little possibility of error. The longitudes of Venus have been derived from Tuckerman's ephemeris by fourth-order Everett interpolation, then rounded to minutes; the expected tabular and rounding errors do not exceed 3′ or 4′. The longitudes of the Mean Sun have been obtained by first calculating the position of the true Sun by linear interpolation from the Tuckerman tables, then applying an equation of center computed on the assumption of point M in Figure I as an equant point; the tabular and rounding errors should not exceed 2′ or 3′. In the cases of both Venus and the Sun, a further error must be allowed for, since the exact time of observation is uncertain by about two hours; however, any error from this source, being about equal for the two bodies, will have a negligible effect on our value for the elongation.

Ptolemy's longitudes for the Mean Sun, it will be noted, are regularly too small by a little over a degree. This is an expected result, correlating with the famous or infamous errors in Ptolemy's equinox observations or pseudo-observations, and with his all-too-low estimate of the amount of precession since the time of Hipparchus.[18] Because of the displacement in Ptolemy's position for the spring equinox, it is to be expected that longitudes of celestial objects given in the *Almagest* for the period of Theon's and Ptolemy's observations will be too small by more than a degree on the average; Peters and Knobel have found the best epoch for Ptolemy's catalogue of zodiacal stars to be A.D. 58, so that by A.D. 138, the epoch Ptolemy supposes for the catalogue, the longitudes of the stars are on the average too small by 66.5′.[19] Thus a fair comparison between the Ptolemaic longitudes and those derived from the Tuckerman ephemeris demands that we add this amount to the Ptolemaic longitudes or subtract it from the Tuckerman figures. The errors in the Ptolemaic elongations remain, of course, unchanged; and it is evident that they make elongations to be equal that in fact differ by 1°25′ in the case of the first pair of observations, and by 25′ in the case of the second pair.

Ptolemy's result for the longitude of the apogee, 55°, differs from the value we have obtained from modern parameters, 57°18′, by only 2°18′, or by the even smaller amount 1°12′ once the adjustment for Ptolemy's displaced equinox is made. The approximation to the positions where the maximum elongations have the least and greatest sums is so good that one may be reluctant to regard the result as an accident. The fact remains that Ptolemy's method is aimed at locating the positions, not where the sums of the opposite maximum elongations are greatest and least, but where these opposite maximum elongations are equal. In this respect Ptolemy misses the mark by about 9°, the correct position being some 64° east of the Ptolemaic equinox, or at longitude 65°5.5′ as previously stated. Also, in the case of all four observations, Ptolemy appears to have missed the times of maximum elongation. Since at maximum elongation the planet and the Sun progress eastward at the same rate, the varying rates of the two bodies as derived from Tuckerman's ephemeris can be compared to find the approximate times of maximum elongation. All four of the Ptolemaic observations prove to be too late by two or three weeks. The discrepancies between the Ptolemaic elongations and those derived from Tuckerman's tables lead one to echo Delambre's surprise on seeing the opposite elongations, calculated from observations that were made without instruments by rough estimations of distances from stars, turn out "si parfaitement égales".[20]

Ptolemy's next step is to obtain a value for the maximum elongation of Venus when the Mean Sun is at longitude 55° (apogee), and also a value for the maximum elongation when the Mean Sun is at longitude 55° + 180° = 235° (perigee). These data are then used to compute the eccentricity of the deferent of Venus, and the relative sizes of epicycle and deferent. The two Ptolemaic observations are compared with the longitudes and elongations derived from Tuckerman's ephemeris in Table II:

Table II

Observation	Ptolemy	Tuckerman	Δ (P − T)
1. 2−3 Epiphi, 13 Hadrian = 20 May 129 (morning)			
Longitude of Venus........	10°36′	12° 2′	−1°26′
Longitude of Mean Sun.....	55°24′	56°26′	−1° 2′
Elongation................	44°48′	44°24′	+0°24′
2. 2−3 Tybi, 21 Hadrian = 18 Nov. 136 (evening)			
Longitude of Venus........	282°50′	282°17′	+0°33′
Longitude of Mean Sun.....	235°30′	236°42′	−1°12′
Elongation................	47°20′	45°35′	+1°45′

In the case of the first of these observations, comparison between the rates of Venus and the Sun in the Tuckerman ephemeris shows that the maximum elongation occurred before 20 May 129, and in fact on the morning of 2 May 129 when the Mean Sun was at longitude 38°46′, the western elongation of Venus then being 44°49′. That this result differs by only 1′ from Ptolemy's value appears to be merely coincidental. In Figure I, the point of observation when the Mean Sun is in 38°46′ is some 18° or 19° clockwise from point A on the Earth's orbit, and it is apparent that the line from this position of the Earth through D, the center of Venus's orbit, passes considerably to the left of point M, or the Mean Sun, so that the greatest western elongation will be considerably less than the greatest eastern elongation. In order to obtain good values for the maximum elongations as they would be seen from point A, one may proceed by the following algebraic and trigonometric steps: first compute the coordinates of the Earth's position with respect to D as origin and the major axis of Venus's ellipse as X-axis; next determine the points of tangency on Venus's ellipse, using equations for the tangents through the Earth's position; finally determine the slopes of the tangents and their inclinations to the line from the Earth through M. Carrying out this calculation for a position of the Earth such that the longitude of the Mean Sun was 56°35′, we obtained the following results: for the greatest western elongation 45°18′, and for the greatest eastern elongation 45°48′. The 30′ difference between these elongations, it has already been noted, is predictable from the displacement of M to one side of the line of centers. The mean of the two elongations is 45°33′, differing by 45′ from Ptolemy's value.

In the case of the second observation, comparison of the rates of Venus and the Sun in the Tuckerman ephemeris shows that the maximum elongation occurred about 9 December, considerably after Ptolemy's date, and was equal to about 47°26′, the longitude of the Mean Sun at this time being 257°23′. In order to obtain good values for the maximum elongations as they would be seen from point P in Figure I, we have recourse once more to the path through the thicket of algebraic and trigonometric calculations. The results, for a position of the Earth such that the longitude of the Mean Sun is 236°35′, are 47°24′ for the greatest western elongation, and 46°53′ for the greatest eastern elongation. The difference is 31′, and is thus once again of the amount and in the direction expected from the displacement of the point M to the left of line PD. The mean of the two values is 47°8′.

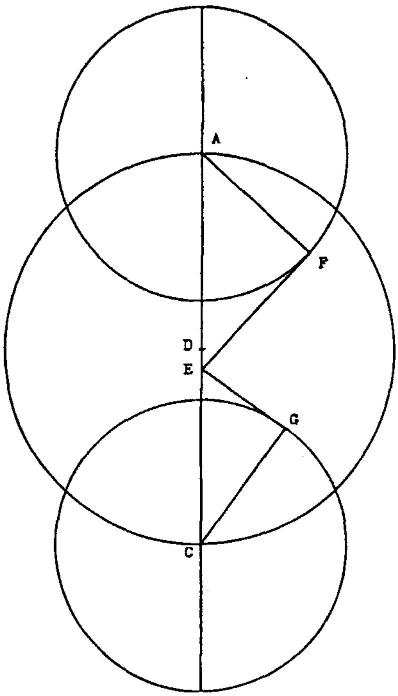

Fig. II

As previously stated, Ptolemy uses his values for the maximum elongations at apogee and perigee to determine the eccentricity and ratio of the epicycle to the deferent. The geometrical relations involved are shown in Figure II. According to Ptolemy, angle AEF is 44°48′, angle CEG is 47°20′, and the angles at F and G are right since the epicycle is circular. Point D is the center of the deferent. By computation Ptolemy finds the eccentricity DE to be 0.021, and the radius of the epicycle to be 0.719, where the radius of the deferent is 1.000. Were we to use instead of Ptolemy's values the mean values of the opposite maximum elongations derived previously from Figure I, namely 45°33′ for angle AEF and 47°8′ for angle CEG, we would find the eccentricity to be 0.0133 and the radius of the deferent to be 0.723. Ptolemy's error of 0.5% in the radius of the epicycle can be regarded as negligible, but his value for the eccentricity is about 58% too large. The eccentricity DE in Figure II is essentially the same as the eccentricity DF in Figure I; and if DF is computed by trigonometric solution of triangle DFS in Figure I, the result is 0.01327, in agreement with the result just obtained from the mean elongations. Ptolemy's erroneous determination of this eccentricity will be one of the premisses leading to his conclusion that the eccentricity of the equant is exactly bisected by the eccentricity of the deferent.

Earlier it was pointed out that the bisected eccentricity, which is a feature common to the Ptolemaic theories of Venus, Mars, Jupiter, and Saturn, emerges in a direct way from the appearances only in the case of Venus. In the case of the superior planets, Ptolemy introduces the bisected eccentricity by fiat. An eccentric deferent with bisected eccentricity is fitted to three observations of the planet when it is in opposition to the Sun; the method of fitting is simply that of trial and error, variations being made in the assumed direction of the line of apsides and in the assumed eccentricity until the theory represents the observations to a close enough approximation. There is no direct test of the bisection. Even for the more fundamental fact that in the theories of these planets Ptolemy uses an equant point separate from the center of the deferent, the justification remains tacit and indirect: presumably theories with simple eccentricity and without the *punctum aequans* had failed to "save the phenomena." We must now see how the equant point and bisected eccentricity emerge out of Ptolemy's data for the maximum elongations of Venus.

In the preceding stage, Ptolemy determined the eccentricity DE of Figure II (or DF of Figure I), and the ratio of epicycle to deferent, from maximum elongations of Venus obtained when the center of Venus's epicycle was at apogee and perigee of the deferent. A natural assumption would be that the center of the epicycle moves uniformly about D, the center of the deferent, and Ptolemy's next step can be imagined as a test of this assumption. If the epicycle moves uniformly on the deferent, its angular speed about D must be exactly equal to the constant angular speed of the Mean Sun about E, the Earth. Therefore, since the epicycle is at perigee (C in Figure II) when the longitude of the Mean Sun is 235°, the epicycle should have moved 90° beyond perigee when the Mean Sun has arrived at longitude $235° + 90° = 325°$. Maximum eastern and western elongations of Venus from the Mean Sun at this longitude permit determination of the position of the epicycle's center on the deferent, and thus answer the question how far it has moved beyond perigee, whether 90° or more or less. The answer turns out to be that it has moved more, so that D is not a center about which the motion is uniform. The observations Ptolemy uses are compared with longitudes and elongations derived from Tuckerman's ephemeris in Table III:

Table III.

Observation	Ptolemy	Tuckerman	Δ (P − T)
1. 2−3 Pharmouthi, 18 Hadrian = 18 Feb. 134 (morning)			
Longitude of Venus........	281°55′	282° 5′	−0°10′
Longitude of Mean Sun.....	325°30′	326°37′	−1° 7′
Elongation................	43°35′	44°32′	−0°57′
2. 4−5 Pharmouthi, 3, Antonine = 18 Feb. 140 (evening)			
Longitude of Venus........	13°50′	14°39′	−0°49′
Longitude of Mean Sun.....	325°30′	326°40′	−1°10′
Elongation................	48°20′	47°59′	+0°21′

In both observations Ptolemy appears to have hit on the time of maximum elongation with little error. In the case of the first observation one can show from Tuckerman's ephemeris that the maximum elongation occurred one day earlier than Ptolemy's date, but the difference in elongation produced by the lapse of this one day is 1′ of arc. In the case of the second observation, a shift of one day either way from Ptolemy's date decreases the elongation. Both observations were made by measuring, presumably by means of an instrument, relatively large angles from fixed stars, in the first case an angle over 59° from Antares and in the second case an angle over 28° from Aldebaran. The error in Ptolemy's longitudes for both these stars is about 1°12′ in A.D. 138,[21] and so is very nearly the same as the error in Ptolemy's equinox and Mean Sun positions. The errors in Ptolemy's latitudes for these stars, namely + 20′ for Antares and + 27′ for Aldebaran, are insufficient to account for the fact that Ptolemy's values for the elongations turn out to be as largely in error as they are. The errors in elongation are such, it will become immediately apparent, that they exaggerate the eccentricity of the equant.

The geometry required to determine the position of the epicycle when the Mean Sun is at 325°30′ is shown in Figure III. Here HQ has been constructed through H perpendicular to AC. Angles FEM and GEM are the greatest eastern and western elongations, respectively, from the Mean Sun, M. Angles FEH and QHE can be determined from the maximum elongations, for FEH is half their sum, and QHE is half their difference. With the first of these angles and the value 0.719 obtained earlier for the radius of the epicycle, one can calculate the length of EH. With the second angle and EH one can then compute QE. Ptolemy gives as his result exactly double the value found earlier for DE; in decimal form, QE = 0.0416 and DE = 0.0208.

If instead of Ptolemy's elongations one used for this computation the elongations obtained from the Tuckerman ephemeris, assuming also the more correct value 0.72333 for the epicycle's radius, the result would be 0.03014. In Figure I this is the distance DX, where X has been located by dropping a perpendicular from M, the equant point, onto the line DF extended. From an earlier calculation we know that DF is 0.01327; thus FX should be 0.03014−0.01327 = 0.01686. The same length can be computed more accurately in another way: FX is given by FM cos 13°42.4′, where FM is equal to the eccentricity of the Earth's orbit (0.01749), whence FX = 0.01699. The small discrepancy between the two results is attributable to the fact that in the calculation from the elongations the line from the Earth to the Mean Sun is not exactly at right angles to the line of centers, or DF extended, this fact being in turn due to the lack of coincidence of Ptolemy's apogee and the longitude of point P in Figure I.

Given the considerable errors in the observations Ptolemy employs, it is difficult to avoid the uncomfortable suspicion that the observational results have been "adjusted" to some extent, to obtain a neatly symmetrical theory with exactly bisected eccentricity. Nevertheless, the question remains whether Ptolemy, in introducing the equant point and in placing it yonder side of the center of the deferent from the Earth, with the distance between equant point and center being of the same order of magnitude as the distance between center and the Earth, is not conveying a genuine empirical discovery of capital importance. The crucial *datum* in the determination of the eccentricity of the equant (EQ in Figure III) is the *difference* of the two maximum elongations, eastern and western, from the Mean Sun when the latter is 90° from the perigee or apogee of Venus's deferent. In order that the point Q in Figure III should be found identical with the point D, given our earlier determination of DE as 0.01327 and assuming observations accurate to within 10′ of arc, it would be necessary that the difference of elongations turn out to be 1°31′ ± 20′. Or if Ptolemy's prior determination of DE as 0.0208 is taken as starting point, the difference of elongations would have to be 2°23′. The difference Ptolemy claims to have found is 4°45′; the difference he should have found is 3°26′; both numbers being considerably larger than the numbers that

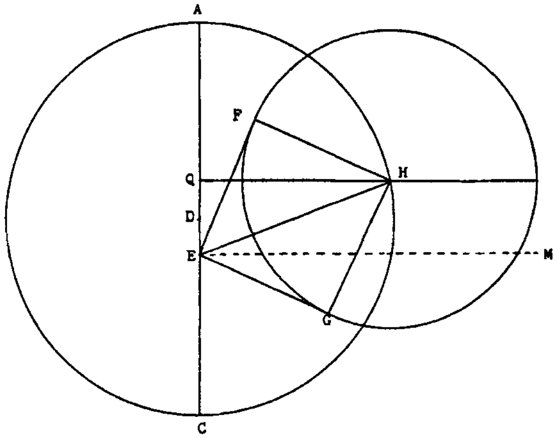

Fig. III

would be required to justify making Q identical with D. It is plausible to suppose that the maximum elongations Ptolemy reports are not the only ones he knew, and that he found himself confronted with a general phenomenon, namely, that for more than one position of the Mean Sun in between perigee and apogee, the difference of the eastern and western maximum elongations had proved larger than a theory with equant point and center of deferent coinciding could allow for. If this supposition is correct, then the principle of the equant or some device having the same effect would have been forced upon him; and given his earlier steps and the character of the phenomenon, one can easily imagine that the equant would present itself as the most obviously available solution of the problem. The *exact* bisection in the Ptolemaic theory, on the other hand, looks like one more piece of evidence pointing to a penchant for tidiness and symmetry, a tendency that goes even to the point of adjusting the observational data so that they would yield a neat and simple theory. Perhaps there is something to be said for using this procedure—although not for concealing one's use of it—if the raw observations were as confusing as we may justly suspect them to have been. That they contained considerable error has been indicated by comparisons between Ptolemy's reported results and the Tuckerman tables; that the problem to be solved does not admit of a strictly symmetrical solution has also been indicated. The precise bisection was better by far than its inventor could have had any valid justification for supposing; better in fact than he could have known even

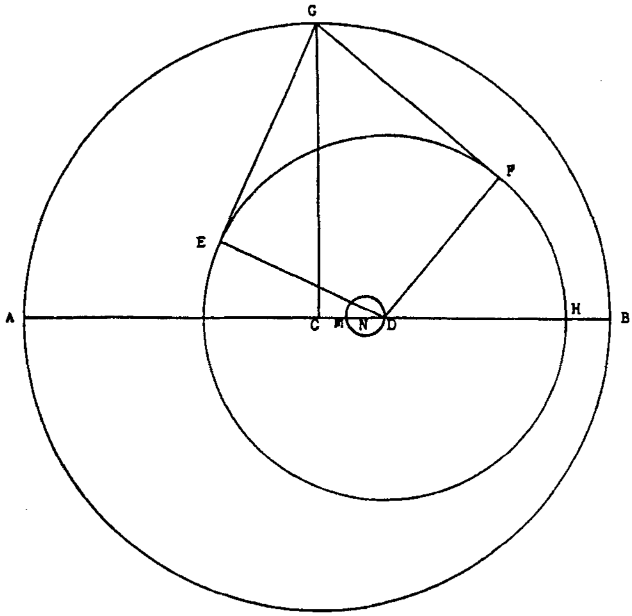

Fig. IV

with very accurate observations, since it is inapplicable to the Ptolemaic deferent of Venus.

What the bisection is strictly applicable to, with an error not exceeding 16″, is the motion of the Earth or Sun; but before this fact can be discerned in the observational data for Venus, the motions that are conflated in the Ptolemaic theory have to be disentangled. It is to Copernicus that one first looks, with the expectation that he will have done this, or at least that he will have wanted to do it. Certainly a major motive for his reformation of planetary theory was to eliminate the unexplained coincidence in Ptolemaic theory that the inner planets have longitudinal periods exactly equal to the Sun's year; and it would appear but a further step in the same direction to demand that the motions of the inner planets be freed from all motions having the Earth's or Sun's period. His theory for the longitudes of Venus is shown in Figure IV. Here the Earth's orbit is circle ABG, with center C; Venus's orbit is circle EHF, and its center moves counter clockwise about the small hypocycle having the center N and radius ND, the angular rate being double that of the Earth, and the timing such that the center is at M when the Earth is a A or B, but at D when the Earth is a quadrant's distance from apogee or perigee as at G. Copernicus accepts Ptolemy's numerical parameters as exact, but finds the distance CD (essentially the eccentricity of the Ptolemaic equant, or DX in Figure I) to have decreased since Ptolemy's time from 0.0416 to 0.0350. Thus the eccentricity CD is no longer bisected by the point M. Copernicus considers this decrease to be due to a periodic motion of the center of the Earth's orbit which has brought it closer to the Sun; he claims to have detected a similarly explicable diminution in the eccentricity of the orbit of Mars. In fact, small decreases in the eccentricities of the orbits of both the Earth and Venus had reduced CD from 0.03026 in Ptolemy's time to 0.02935 in A.D. 1530. Thus the eccentricity given by Copernicus is somewhat less in error than the Ptolemaic value. The more fundamental fact is that Copernicus retains the essential structure of the Ptolemaic theory. He assumes symmetry about the line of centers; and having transformed Ptolemy's solar theory, with its uniform motion on an eccentric circle, into a theory of identical form for the Earth, he is forced to accommodate the equant point in Ptolemy's theory of Venus by the introduction of the strange little hypocycle. In accepting the Ptolemaic observations as precisely accurate, and in following the Ptolemaic practice of using the Mean Sun rather than the true Sun as point of reference, Copernicus found himself forced into a certain opportunism of explanation, similar to that which Ptolemy confesses to in his introduction to the theory of the latitudes (*Almagest* XIII, 2). The decision for the hypocycle rather than for a mere linear oscillation (such as Copernicus uses in his theory of the latitudes), and the decision for counterclockwise rather than clockwise motion on the hypocycle, can hardly have been based on clear observational evidence; they constituted a highly arbitrary solution of a difficulty, a special device that was not and could not be (in the then state of the observational art) tested in all its consequences.

It is Kepler who finally teases apart the entangled causes, and arrives at a theory for Venus free from such narrowly arbitrary postulations. The Keplerian theory of Venus is exactly like the Keplerian theories of the other circumsolar planets: there is an eccentric elliptical orbit, with the Sun occupying one focus, and the planet moves in accordance with Kepler's area rule. Kepler finds the eccentricity to be 0.00692, and the longitude of the perigee in A.D. 1600 to be 121°14′; extrapolating from modern parameters we would expect these numbers to be 0.00697 and 125°55′

respectively. The error of 4°41′ in the perigee is to be expected as a result of the excessive value Kepler gave to the eccentricity of the Earth's orbit, 0.01800 instead of 0.01688 (in effect, the lengthening of SF in Figure I pushes S away from F and rotates the line SD clockwise). Kepler's orbital elements will, of course, be subject to later refinement. One must note, on the other hand, that they were not originally arrived at by a process that can be accurately described as the refinement of an earlier theory; the essential step was rather the introduction of a new pattern, borrowed from the planetary theory to which Kepler had been led by his studies of Mars. Gone is the assumption of symmetry about the line of centers; gone the intrusion into the theory of Venus of motions synchronous with the Earth's motion. Kepler separates what belongs to Venus and what to the Earth in a new way; the theory of each planet is symmetrical about a line of apsides passing through the Sun, and each theory describes the dynamic relation of a planet to the Sun. At the root of this as of other phases of the Keplerian revolution is Kepler's hunch concerning the causal efficacy of the Sun. A new way of seeing the world as connected system is operative here, one that was utterly alien to the minds of such contemporary astronomers as Lansberg and Longomontanus. The new theory of Venus was not merely an inductive outcome of the available observations of Venus, and indeed is not imaginable as such; the transits of the planet across the face of the Sun, which can alone provide direct information as to its heliocentric longitudes, are too infrequent (the first such transit was observed by Horrox in 1639). Here theory had outrun fact.

Mercury

As previously indicated, the phenomena of Mercury are different from those of Venus, and more complicated. Kepler mentions one difference in his letter to Magini of June 1601, and again in his letter to Maestlin of 10/20 December 1601, when he says that, whereas the apogee of Venus and that of the Sun are almost conjoined, the apogee of Mercury is closer to the perigee than to the apogee of the Sun.[22] We can put the matter in perhaps more graspable form by saying that the center of Venus's orbit is closer to the Sun, and the center of Mercury's orbit is farther from the Sun, than the center of the Earth's orbit; all three centers being toward the same side of the Sun, confined within an angle of about 60°. It will be seen shortly that this arrangement brings it about that in the Ptolemaic theory of Mercury the equant point is closer than the center of the deferent to the Earth.

A further complication in the Ptolemaic theory arises from the additional feature that the center of the deferent is made mobile, moving on a small circle in such a way as to bring the epicycle of Mercury, when it is in between apogee and the opposite point of the deferent, closer to the Earth than a fixed deferent would permit. In fact, the epicycle is brought closest to the Earth not at the point 180° from apogee but at two points just about 120° to either side—the double perigee of Ptolemaic and Copernican theory. Now at the time of his letters to Magini and Maestlin in 1601, Kepler believed the orbits to be perfectly circular. If the orbits are postulated to be circular, and if the simplifying assumption is made that they occupy a single plane—the 7° inclination of Mercury's orbit to the plane of the ecliptic thereby being ignored—it turns out that the alleged phenomenon of the double perigee of Mercury is underivable. It is therefore understandable that Kepler, while remaining doubtful as to the fact of the phenomenon, considered the possibility that the inclination of Mercury's orbit might account for this feature of the Ptolemaic and Copernican theories.[23] In fact, the boreal node of Mercury is fairly close to the perigee of Mercury; in A.D. 140 the difference in the longitudes of the two points was 22° (it has since increased to about 29°). This means that the projection of the orbit onto the ecliptic is narrower in the middle longitudes between aphelion and perihelion than it would otherwise be. Consequently, the sums of opposite maximum elongations, for positions of the Mean Sun near apogee or the opposite point of the ecliptic, are reduced from what they would be if the inclination were zero, relative to the sums for intermediate points. The shortening of the radius vector by projection onto the ecliptic, or *curtatio* as Kepler called it, leads to a reduction of maximum elongations of about 11′ or 12′ at most.

An effect of the same character but of greater magnitude is produced by the oval shape of the orbit, which reduces the elongations in the middle longitudes by as much as half a degree. From the moment that Kepler knew the Martian orbit to be oval, he assumed the Mercurial orbit to be oval as well. His recognition that the ellipticity could imply a double perigee, and his simultaneous doubt of the truth of the latter, is expressed in a letter to David Fabricius of October 1605; he is responding to Fabricius's proposal of a theory for the latitudes of Mercury:

> Here you call me into your labyrinths of Mercury and Venus. It is not permitted to follow: you will weary yourself without reason, assuming the Ptolemaic opinions, so many of which are found to be false. To me it suffices concerning Venus and Mercury to know from observations that the inclinations of the planes (which are epicycles for Ptolemy) to the plane of the ecliptic are constant ... The small libratory circles ... fall of their own accord, once the eccentricity of the Earth is bisected. From the epicycles are made eccentrics, each with its equant ... The *triangulatio* of Mercury [Kepler is referring to the three Ptolemaic apses, namely one apogee and two perigees 120° apart] is suspect, for the whole of the observed quantity that persuaded Ptolemy of the triangulatio, and something more, is attributable to the uncertainty of the observation. And yet if from the epicycle an eccentric is made, the eccentricity will be large, and the ovalness very marked—causes which could lead Ptolemy to the triangulatio. I would add also this, concerning which I frequently ask, why today are not the sums of the elongations equal to the ancient values?...[24]

Is Kepler right to suppose that the observational error in the data Ptolemy uses exceeds the difference that is put forward as justifying the double perigee? Is there in fact a double perigee— or more exactly, are there phenomena that would lead Ptolemy with his assumptions to establish a double perigee, as Kepler seems to suggest in the just-quoted passage and as he comes to assert in the *Epitome astronomiae Copernicanae*?[25] Is Kepler right when, in the same passage of the *Epitome*, he suggests that Ptolemy's observational data were "adjusted" to yield a symmetrical theory? In what way, and to what extent, does the Ptolemaic artifice mirror fact? And Copernicus's reformulation of the Ptolemaic theory: does it improve matters? In answering these questions, we can bring into focus the advance and the innovation in the Keplerian *instauratio.*

As with Venus, so with Mercury, the fundamental Ptolemaic data are maximum elongations from the Mean Sun. Mercury, like Venus, has both heliacal and zodiacal anomalies. Once more, Ptolemy accounts for the heliacal anomaly by counterclockwise motion on an epicycle. The zodiacal anomaly, however, requires variable eccentricity of the deferent as well as an equant point. The initial problem in setting up all the Ptolemaic planetary theories is to separate the two anomalies, and following his procedure in the case of Venus, Ptolemy does this for Mercury by assuming symmetry and seeking to locate a line of apsides. The pairs of opposite maximum elongations that Ptolemy uses for this purpose are compared with the results of computation from the Tuckerman ephemeris in Table IV (the Tuckerman positions of Mercury are computed by fourth order Everett interpolation; the positions of the Mean Sun are computed as previously described in the case of Venus).

Table IV

Observation	Ptolemy	Tuckerman	Δ (P − T)
1. 16−17 Phamenoth, 16 Hadrian = 2 Feb. 132 (evening)			
Longitude of Mercury......	331° 0′	330°36′	+0°24′
Longitude of Mean Sun.....	309°45′	310°50′	−1° 5′
Elongation...............	21°15′	19°46′	+1°29′
2. 18−19 Epiphi, 18 Hadrian = 4 June 134 (morning)			
Longitude of Mercury......	48°45′	51°14′	−2°29′
Longitude of Mean Sun.....	70° 0′	71° 7′	−1° 7′
Elongation...............	21°15′	19°53′	+1°22′
3. 20−21 Epiphi, 1 Antonine = 4 June 138 (evening)			
Longitude of Mercury......	97° 0′	97°18′	−0°18′
Longitude of Mean Sun.....	70°30′	71°39′	−1° 9′
Elongation...............	26°30′	25°39′	+0°51′
4. 18−19 Phamenoth, 4 Antonine = 2 Feb. 141 (morning)			
Longitude of Mercury......	283°30′	285°54′	−2°24′
Longitude of Mean Sun.....	310° 0′	311° 9′	−1° 9′
Elongation...............	26°30′	25°15′	+1°15′

Refraction is no doubt partly responsible for Ptolemy's exaggerated elongations, but it is not the only source of error in these observations. Comparison of rates of Mercury and the Sun in the Tuckerman ephemeris indicates that the second and third observations listed above were too late for the maximum elongation, the second by about five days and the third by about four days. The first observation was about a day and a half late, and the fourth was made at very nearly the right time. The four maximum elongations, deduced from the Tuckerman ephemeris for the times thus found, are: 19°51′, 21°19′, 25°54′, and 25°19′. The first two numbers, which according to Ptolemy should be equal, differ roughly by a degree and a half; the third and the fourth, which also should be equal, differ by about half a degree. An excursion into geometry will now confirm these results, and show that the line of symmetry Ptolemy claims to have found does not exist.

Figure V shows the orbits of Mercury and the Earth: S is the Sun, F the center of the Earth's orbit, M the equant point for the Earth's motion (and thus the second focus of the Earth's elliptical path), and D the center of Mercury's orbit. The eccentricities are exaggerated to show the geometry at the center; hence the orbits are made to appear less concentric than they are in fact. The eccentricity of the orbit of Mercury, computed by extrapolation from present-day values, is found to have been 0.205265 in A.D. 140; if the mean radius of the Earth's orbit is taken as unity, so that the mean radius of Mercury's orbit becomes 0.387098, the eccentricity of Mercury becomes 0.0794577; and when this length is projected onto the ecliptic through the angle of inclination of the orbit (6°58′ in A.D. 140), the final result is a value for SD in the diagram, namely 0.078868. The eccentricity of the Earth's orbit is taken, as earlier, to be 0.01749. Also as previously, we take the longitude of the Earth's perihelion in A.D. 140 to be 71°0′. The longitude of Mercury's perihelion is calculated to be 48°33′.

Using these numbers and the analytic geometry of the ellipse, we can compute maximum eastern and western elongations for any stipulated longitude of the Mean Sun, just as in the case of Venus.[26] By such computation, the four maximum elongations corresponding to the Mean Sun positions stipulated in Table IV are found to be: 19°51′, 21°39′, 25°43′, 25°19′. The second and third of these differ from the maximum elongations obtained previously from the Tuckerman ephemeris for the reason that the Mean Sun positions are significantly different in the two cases. The new figures confirm the conclusion that Ptolemy's observations fail to locate an axis of symmetry: the opposite maximum elongations are unequal. Thus Ptolemy's apogee of the deferent, located at about 11° by averaging the two positions of the Mean Sun in observations # 1 to # 4 of Table IV, is not such that the maximum eastern and western elongations are there equal. The geometry shows that, for the Mean Sun at longitude 11°5′, the maximum eastern elongation is 22°27′, the maximum western elongation 24°59′.

For what position of the Mean Sun near Ptolemy's apogee will the opposite maximum elongations be equal? Insofar as the orbit of Mercury approaches circularity, we could expect this position to be on the line in Figure V that passes through D, the center of Mercury's orbit, and M, the equant point for the Earth's motion. By solution of triangle SDM, angle SDM is found to be 16°1′, whence the proposed longitude of the

Mean Sun is 48°33′ − 16°1′ = 32°32′. From this position the maximum eastern elongation is found to be 23°45′, and the maximum western elongation 23°55′; the 10′ difference is a result of the ellipticity of the orbit (the *curtatio* here decreases, rather than increases, the difference). By the time the Mean Sun has moved on to a longitude of 40°, the steadily increasing maximum eastern elongation has exceeded the maximum western elongation by about 54′. The opposite maximum elongations are equal, we may estimate, when the Mean Sun has a longitude of about 34°. Ptolemy's guess thus misses the mark by about 22°.

Ptolemy's next step, as in the case of Venus, is to determine the eccentricity and relative size of Mercury's epicycle from observations of maximum elongations when the Mean Sun is at apogee and the opposite point of the ecliptic. The two observations used, compared with the corresponding longitudes and elongations derived from Tuckerman's ephemeris, are given in the following table.

Table V

Observation	Ptolemy	Tuckerman	Δ (P − T)
1. 14–15 Athyr, 19 Hadrian[27] = 3 October 134 (morning)			
Longitude of Mercury......	170°12′	170°45′	−0°33′
Longitude of Mean Sun.....	189°15′	190°23′	−1° 8′
Elongation................	19° 3′	19°38′	−0°35′
2. 19–20 Pachon, 19 Hadrian[27] = 5 April 135 (evening)			
Longitude of Mercury......	34°20′	34°25′	−0° 5′
Longitude of Mean Sun.....	11° 5′	12°13′	−1° 8′
Elongation................	23°15′	22°12′	+1° 3′

Comparison of rates in the Tuckerman ephemeris indicates that Ptolemy has missed the time for maximum elongation, especially in the first observation but also in the second. In the case of the first observation, the maximum elongation appears to have occurred about four days earlier, on the morning of 29 September; the value of the elongation at that time was 20°15′. In the case of the second observation, the maximum elongation appears to have occurred about a day and a half earlier; the value of the elongation was then about 22°21′. Computations from the geometry of Figure V confirm the fact that the elongations calculated from the Tuckerman tables for the times given by Ptolemy are less than maximal: with the Mean Sun in 189°15′, the maximum western elongation proves to be 20°16′, and with the Mean Sun in 12°19′, the maximum eastern elongation proves to be 22°27′. There is a further, serious difficulty. Since Ptolemy assumes symmetry about the line of apsides that he has established, he supposes that the maximum eastern and western elongations when the Mean Sun is in 189°15′ are equal, and similarly that the maximum eastern and western elongations when the Mean Sun is in 11°5′ are equal. But in each case the supposition is incorrect, as indicated in the following results computed from the geometry of Figure V:

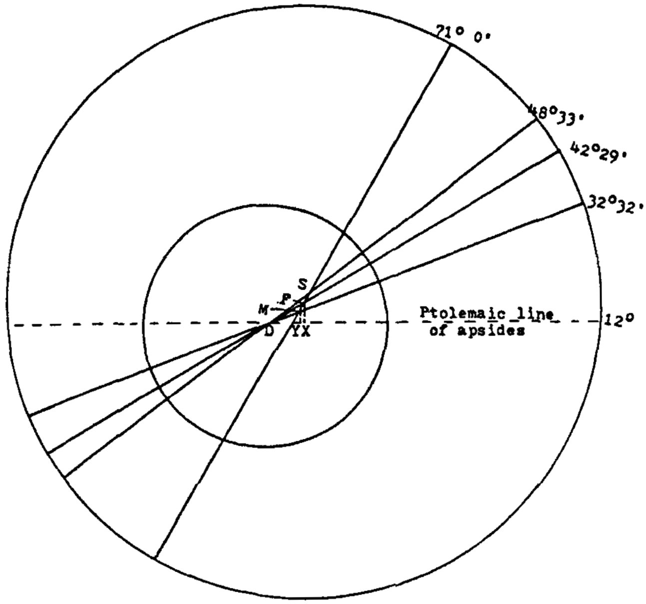

Fig. V

It is the mean value in each case that informs us concerning the apparent size of the epicycle as "seen" from the Earth. Hence Ptolemy's value for the first elongation is more than two degrees too small, and his value for the second elongation is about half a degree too small.

Ptolemy's diagram for computation of eccentricity and relative size of epicycle is reproduced in Figure VI. His results, translated into decimal

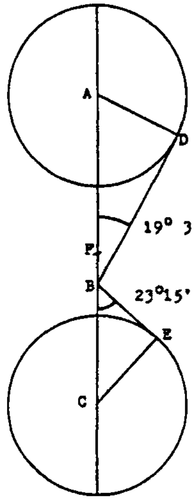

Fig. VI

form, are that the eccentricity, FB/AF, is 0.095, and that the radius of the epicycle in relation to half of line AC, or CE/AF, is 0.357. If as basis of the computation the mean values from Table VI are used instead of Ptolemy's values, the results are: for the eccentricity, 0.0543, and for the radius of the epicycle, 0.3804. Ptolemy's value for the eccentricity is nearly 75% too high; his value for the radius of the epicycle about 6% too low.

Ptolemy's third step again parallels his procedure for Venus: he exam-

Table VI

	Ptolemy	Result Calculated from Figure V	
Mean Sun in 189°15′			
Greatest eastern elongation.........	......	22° 3′	Mean Value = 21° 9′
Greatest western elongation........	19° 3′	20°16′	
Mean Sun in 12°19′			
Greatest eastern elongation.........	23°15′	22°27′	Mean Value = 23°43′
Greatest western elongation........	......	24°59′	

ines the maximum elongations when the Mean Sun is a quadrant's distance from apogee, with the aim of locating the equant point on the line of apsides. The observations he uses are compared with the results of calculation from the Tuckerman ephemeris in Table VII.

The first observation, which is due to Theon, appears to have been made very near the time of maximum elongation; calculation from the geometry of Figure V, with the Mean Sun in 101°8′, yields a maximum eastern elongation of 26°16′, within 1′ of the elongation derived from Tuckerman's ephemeris. In the case of the second observation, however, the maximum elongation occurred some three days after Ptolemy's date; from the Tuckerman ephemeris it appears to have been about 20°21′.

Table VII

Observation	Ptolemy	Tuckerman	Δ (P − T)
1. 18−19 Mesore, 14 Hadrian = 4 July 130 (evening)			
Longitude of Mercury......	126°20′	127°23′	−1° 3′
Longitude of Mean Sun.....	100° 5′	101° 8′	−1° 3′
Elongation................	26°15′	26°15′	0° 0′
2. 20−21 Mesore, 2 Antonine[28] = 5 July 139 (morning)			
Longitude of Mercury......	80° 5′	81°39′	−1°34′
Longitude of Mean Sun.....	100°20′	101°27′	−1° 7′
Elongation................	20°15′	19°48′	+0°27′

Calculation from the geometry of Figure V, with the Mean Sun in 101°8′, yields a maximum western elongation of 20°25′. Ptolemy's result differs from this by only 10′.

To determine the position of the equant point, Ptolemy uses the diagram reproduced in Figure VII. Here B is the Earth, the dashed line BM at right angles to AC is directed toward the Mean Sun, and the two angles KBM and MBL are the two elongations of Table VII. The reason why the center of the deferent is not placed on the line of apsides will become apparent at the next stage. It is assumed that the epicycle remains of the size determined at the preceding step. With these things given, Ptolemy shows that BG = 0.0474, or just half the eccentricity previously determin-

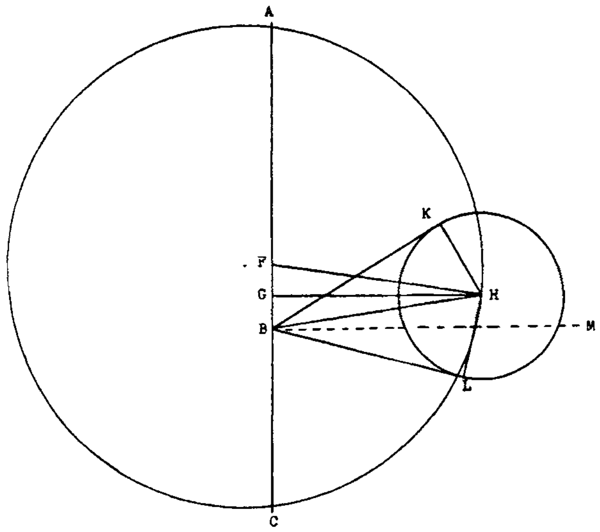

Fig. VII

ed, 0.095. If in this computation we use instead of Ptolemy's values the two elongations determined from the geometry of Figure V, namely 26°16′ and 20°25′, along with the radius of the epicycle as previously determined, namely 0.3804, the result for BG is 0.0490, which is close to Ptolemy's value but considerably more than half our previous value for the eccentricity, 0.0543.

These results can be calculated in a more direct way from the geometry of Figure V. Here DX, the projection of DF onto the Ptolemaic line of apsides, should be and is in fact found trigonometrically to be equal to 0.0543. DY, the projection of DM onto the Ptolemaic line of apsides, turns out to be 0.0453, something less than the value 0.0490 just obtained from elongations; the difference depends on the extreme sensitivity of the determination to the position of the Mean Sun, and on the fact that in the observations one is "viewing" not a circular epicycle but an elliptical orbit, and this from an angle oblique to the axis of the ellipse. In the determination of the eccentricity of the deferent, DX, the difficulty does not arise, since at both apogee and the point opposite the opposite elongations are averaged, and the ellipticity then affects the results for the two positions in very nearly the same way. In any case, we have to conclude that Ptolemy's neat bisection, which makes DY just half of DX, was

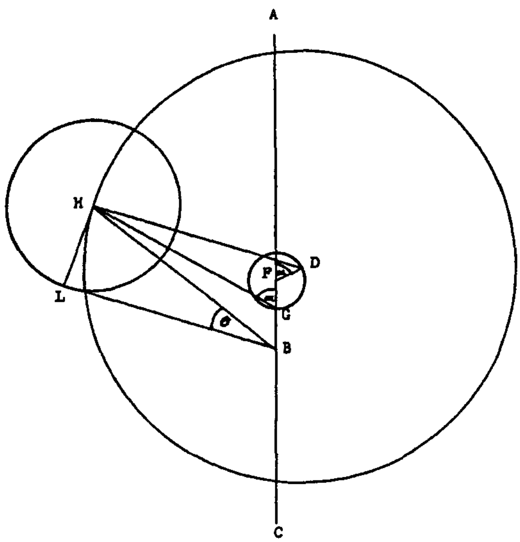

Fig. VIII

obtained either by a prodigious stroke of luck or else by deliberate engineering, that is, suiting the observations to the theory rather than vice versa.

In a general way, however, the Ptolemaic result informs us (in Ptolemaic terms) of an empirical truth: the eccentricity of the equant is less than the eccentricity of the deferent. This first major difference between the Ptolemaic theories of Mercury and Venus evidently arises from a marked difference in the distances between the centers of the orbits and the Sun: the distance from the center of Venus's orbit to the Sun is only about one-third of the distance from the center of the Earth's orbit to the Sun, while the latter distance is only some two-ninths of the distance from the center of Mercury's orbit to the Sun. In each of the two theories the equant point is or should be a projection of the equant point of the Earth's orbit (M in Figures I and V) onto the assumed line of apsides. Roughly, in the arrangement of these various points with respect to one another, the Ptolemaic theories reflect the way things were and are.

One further, major difference between the Ptolemaic theories of Mercury and Venus remains to be examined: the fact that the center of Mercury's deferent is not fixed like that of Venus, but is made to move clockwise on a small circle with the angular speed of the Mean Sun. That some such device is required by the data Ptolemy gives us can be deduced from the Ptolemaic elongations already recorded in Table VII, and the geometry of Figure VII. According to Ptolemy, angle KBH in Figure VII is 23°15′, and KH is 0.357; then BH = KH/sin 23°15′ = 0.904. Now since Ptolemy finds BG = GF, where F is the midpoint of the line of apsides, it follows that FH is also equal to 0.904. But for a fixed center of the deferent we would expect this length to be equal to FA, that is, 1.000.

Is the phenomenon genuine? We repeat the calculation using the values obtained geometrically from Figure V, namely KH = 0.3804, angle KBH = 23°20.5′, BF = 0.0543, and BG = 0.0490. The result is that BH = 0.9601, and FH = 0.9589. The effect is not as marked as Ptolemy claims, but it is nevertheless present. If an epicycle of fixed size is to be assumed, it is not possible to employ a deferent of both fixed size and center. Ptolemy chooses to make the center mobile, moving it in such a way as to bring the epicycle closer to the Earth. On the other hand, when the Mean Sun is at 281°, once more a quadrant's distance from apogee but now on the opposite side of the zodiac, this mechanism works in the wrong direction, bringing the epicycle about as much closer as it needs to be carried farther away.

The peculiar mechanism that Ptolemy devises for Mercury is shown in Figure VIII. Here B is the Earth, G is the equant point, and D is the center of the deferent, moving on the small circle with center F and radius FD. As GH rotates counterclockwise at the rate of the Mean Sun, FD rotates clockwise at the same rate. DH is of constant length; if we hold to our earlier stipulation of AF as the unit of distance, and further set FD = e, then HD = 1 − e. According to Ptolemy's reported findings, as previously noted, BG = FG = e. Given these assumptions, it follows that the epicycle is brought closest to point F when angle AGH is precisely 120° or 240°. This is the reputed phenomenon of "Mercury twice perigee." Does it occur?

If Mercury were to be well-behaved like Venus, exhibiting only a single perigee, we would expect to find this perigee 180° from apogee; that is, at a longitude of 11° for Ptolemy or a little more than 12° if correction is made for Ptolemy's misplaced equinox. With the Mean Sun at longitude 12°, then, we would expect the sum of Mercury's greatest eastern and western elongations to reach its largest value. According to Ptolemy, this sum is 46°30′; the geometry of Figure V, by contrast, implies that it is 47°26′. For the two perigee positions of the Ptolemaic theory, the values of the greatest eastern and western elongations as stated by Ptolemy have already been listed in Table IV. These values, together with their sums, are compared with the values for the same positions deduced from Figure V in Table VIII.

In the case of both alleged perigees, the sum of opposite maximum elongations as calculated from Figure V proves less than the sum calculated in the same way for the Mean Sun at longitude 12°. The perigee asserted for longitude 310° is especially untenable. The only plausible explanation is that the two Ptolemaic perigees are artifacts of a theory. We can still ask: are there two perigees or perhaps even more, at different longitudes from those assigned by Ptolemy? As Kepler was aware, the ellipticity of the orbit of Mercury, and the reduction of the radius vector to the ecliptic, could conceivably lead to such a phenomenon.

Table VIII

	According to Ptolemy	Calculated from Figure V	Δ (P − C)
1. First Perigee (Longitude of Mean Sun = 310° according to Ptolemy, 311° when corrected)			
Greatest eastern elongation..	21°15′	19°51′	+1°24′
Greatest western elongation .	26°30′	25°19′	+1°11′
Sum.....................	47°45′	45°10′	+2°35′
2. Second Perigee (Longitude of Mean Sun = 70° according to Ptolemy, 71° when corrected)			
Greatest eastern elongation..	26°30′	25°43′	+0°47′
Greatest western elongation .	21°15′	21°39′	−0°24′
Sum.....................	47°45′	47°22′	+0°23′

An answer to this question satisfactory for present purposes (we are concerned not with fine structure but with features detectible by means of observations good to within about 10′ of arc) can be obtained by computing the sums of opposite maximum elongations for a series of positions

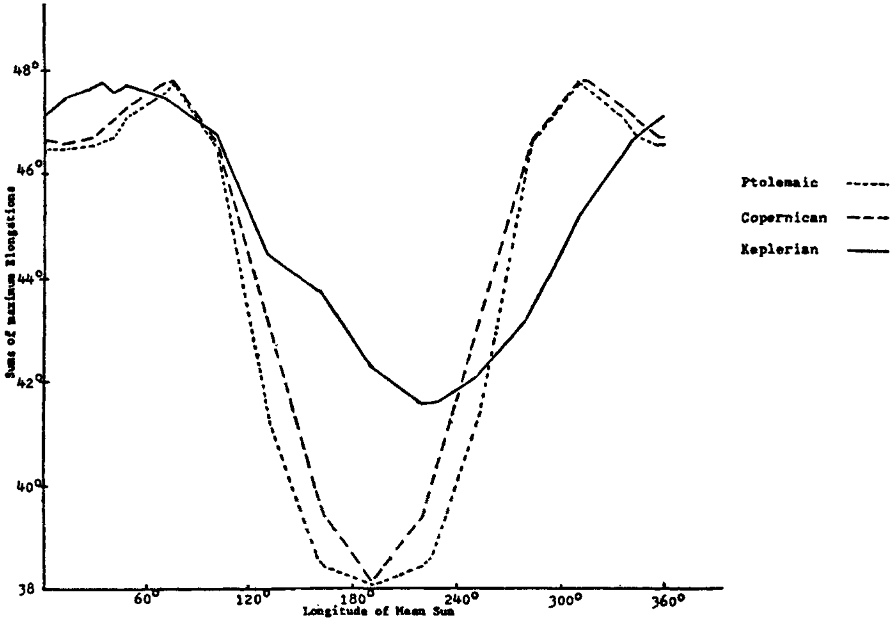

Fig. IX. Plot of Sums of Maximum Elongations of Mercury: Ptolemaic, Copernican, & Keplerian.

around the zodiac, given the Keplerian orbits of Mercury and the Earth as shown in Figure V. The results are shown in the solid-line graph of Figure IX. A slight dip appears in the peak of the graph. It emerged only at the final stage of the calculations, with the introduction of the *curtatio*; as it amounts to no more than 7′ or 8′, it will here be ignored. This dip aside, the graph presents but a single maximum. There is thus but one perigee.

Figure IX also includes a graph of the predictions of the Ptolemaic theory. These predictions have been generated from the mechanism of Figure VIII by means of a series for the sine of the angle between BH and the tangent from B to the epicycle. The variable here is the distance between the Earth B and the center H of the epicycle.[29]

A third graph included in Figure IX shows the sums of greatest eastern and western elongations predicted from the Copernican theory. The Copernican mechanism can be explained with the assistance of Figure X. Just as in the case of Venus, Copernicus here eliminates the equant, in-

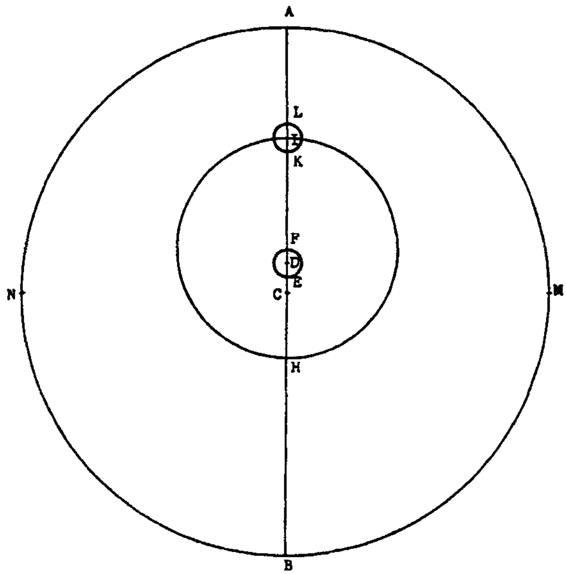

Fig. X

troducing a hypocycle to perform its function; in Figure X the center of the hypocycle is D, and its radius is DF. The center of Mercury's orbit completes its journey round the hypocycle in precisely half the Earth's year; it is at F when the Earth is at A or B, and at E when the Earth is at M or N, midway between A and B. This rotation, which is set up precisely in order to bring the center of Mercury's orbit to the positions required by the Ptolemaic observations when the Earth is at A, M, B, and N, has additional effects. While the Earth is passing from A to N, the orbit of Mercury is moved to the left as well as down; while the Earth is passing from N to B, the orbit is moved to the right as well as up. As we have already seen, the reported Ptolemaic observations require that, for a position of the Mean Sun (or Earth) midway between the two apses, the epicycle (or orbit) must be closer to the Earth than a fixed deferent permits—assuming the epicycle (or orbit) to be of constant size. In the Copernican scheme the Ptolemaic deferent is replaced by the fixed orbit of the Earth; in order to achieve the required result Copernicus therefore makes the orbit itself variable in size, its radius subject to a libratory decrement and increment in accordance with a harmonic oscillation of amplitude IK. The radius has its least value when the Earth is at A or B, and its greatest

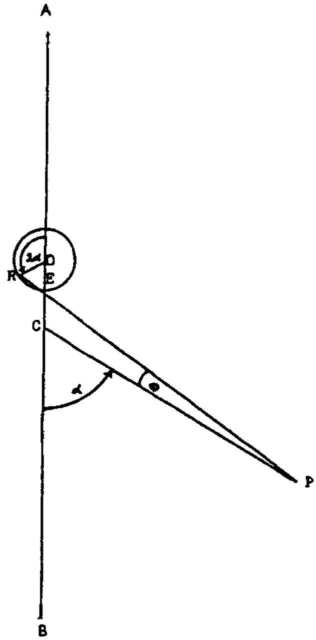

Fig. XI

value when the Earth is at M or N; the period of the oscillation is thus, like the period for the rotation of the center of the orbit, half a year. The calculation of the maximum elongations predicted by the theory proceeds on the basis of Figure XI, where the letters A, B, C, D, E have the same significance as in Figure X, and where P is the Earth, R the momentary center of Mercury's orbit, and α the Mean Sun's motion from aphelion. First the length PR must be computed, then the angle of elongation of Mercury from the center of its orbit, and finally the angle φ between the center of Mercury's orbit and the Mean Sun (C) as seen from the Earth.[30]

The greatest eastern and western elongations which are summed to give the points plotted in Figure IX are plotted separately in Figure XII, the greatest eastern elongations as positive ordinates and the greatest western elongations as negative ordinates. It is apparent that the Ptolemaic and Copernican predictions of maximum elongations do not exactly agree, and that the disagreement can be as high as 65′, as between the greatest eastern elongations when the Mean Sun is at 134°, and between

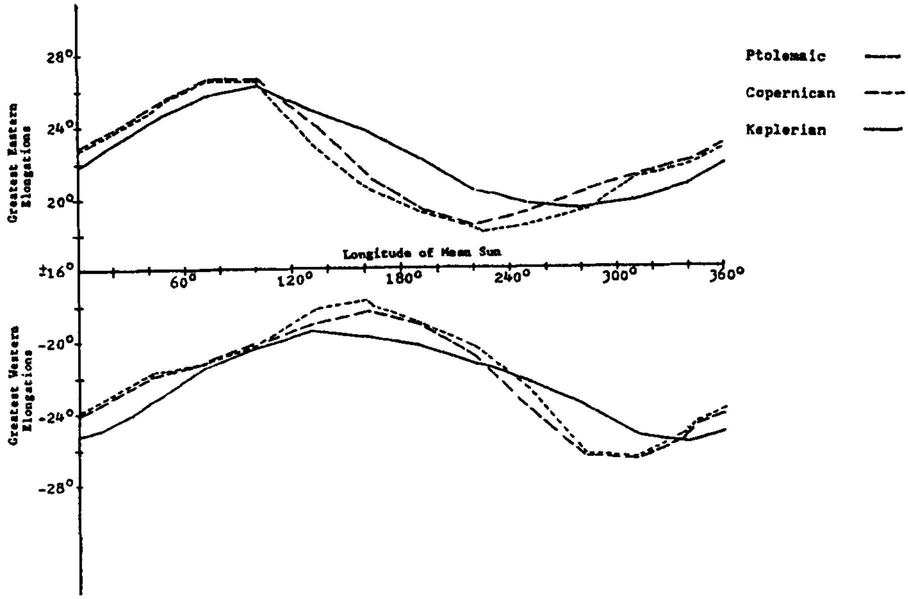

Fig. XII. Plot of Maximum Eastern & Western Elongations of Mercury: Ptolemaic, Copernican, and Keplerian.

the greatest western elongations when the Mean Sun is at 251°. Meanwhile, the two theories agree more nearly with each other than either agrees with the geometry of Figure V. Thus the Ptolemaic prediction of the greatest eastern elongation when the Mean Sun is in 160° or 190° is over 3° less than the value deduced from Figure V.

On the basis of three observations, one made by Bernard Walther in 1491 and two others made by Johann Schöner in 1504, Copernicus concludes that the longitude of Mercury's apogee in his own time is 238½°, "for it was not possible," he says, "to take it as less without prejudice to the observations" (*De revolutionibus* V, 30). This result constitutes a marked improvement over Ptolemy's determination. If we extrapolate modern values for the eccentricities and perihelia of the Earth and Mercury backwards to A.D. 1500 and use once more the geometrical relations in Figure V to compute the directions of the lines FD and MD, we find for the first 243°34′ and for the second 233°33′. For a position of the Mean Sun along the line FD in the direction of apogee, as previously explained, the sum of the opposite maximum elongations is least; for a position of the Mean Sun on line MD extended, the opposite maximum elongations are equal. Copernicus's value for the longitude of the apogee falls just midway between these two values, and so could not be noticeably improved upon. Ptolemy's value for the longitude of the apogee, we may recall, fell 20° clockwise of MD.

Copernicus's improvement in the longitude of the apogee, however, does not go very far towards reducing the large errors in maximum elongations that follow from his assumed circular motions and numerical constants for Mercury. His assumption that this mechanism not only fits the Ptolemaic observations but also remains valid in his own time is based on a faith in earlier astronomers: "I think we must grant that the commensurability of the circles has remained from Ptolemy's time to now, since in the case of the other planets the good authorities who preceded us are not found to have been mistaken here" (*De revolutionibus*, V, 30). In fact, both the Ptolemaic and Copernican theories could have been very considerably improved, insofar as their predictions of maximum elongations are concerned, by a better choice of numerical constants. Thus to obtain a better Ptolemaic theory we can put the perigee for A.D. 140 at 33°40′, and take the magnitude of the epicycle as 0.377, the eccentricity of the deferent (BF in Figure VIII) as 0.0643, the eccentricity of the equant (BG in Figure VIII) as 0.0476, and the radius of the small hypocycle (FD in Figure VIII) as 0.0098. The maximum elongations implied by these constants for perigee, apogee, and the quadrants are compared with the maximum elongations deduced from Figure V in the following table.

Improvement in Ptolemy's numerical constants can thus considerably reduce the errors in predictions of the magnitude of maximum elongations. Such improvements in the theory, however, cannot eliminate errors of as much as five days in the predictions of the *times* of maximum elongation, since the Ptolemaic theory gives the planet a uniform motion on its epicycle, while its actual orbital motion is markedly non-uniform. This non-uniformity affects geocentric longitude still more when the planet is at positions other than maximum elongation, and its largest observable effect occurs when Mercury is in transit across the face of the Sun. Transits of Mercury occur at irregular intervals, about fourteen of them on the average per century; the duration of any one transit does not exceed about four hours. A theory's successful prediction of one such transit could well be an extraordinary piece of luck. Prediction of three such

Table IX

	Deduced from Figure V	Implied by New Constants	Δ
1. Elongations from Mean Sun in 33°40′ (perigee)			
Greatest Eastern Elongation........	23°45′	23°46′	1′
Greatest Western Elongation.......	23°50′	23°46′	4′
Average..........................	23°47′	23°46′	1′
2. Elongations from Mean Sun in 213°40′ (apogee)			
Greatest Eastern Elongation........	20°38′	20°45′	7′
Greatest Western Elongation.......	20°54′	20°45′	9′
Average..........................	20°46′	20°45′	1′
3. Elongations from Mean Sun in 123°40′ (quadrant)			
Greatest Eastern Elongation........	25°44′	25°22′	22′
Greatest Western Elongation.......	19°59′	19°49′	10′
4. Elongations from Mean Sun in 303°40′ (quadrant)			
Greatest Eastern Elongation........	19°42′	19°49′	7′
Greatest Western Elongation.......	25° 4′	25°22′	18′

transits to within a few hours, where the older theories were missing the mark by several days, would not establish the theory as a straightforwardly empirical truth, but even to the theory's most adamant opponents, such success could hardly fail to give pause. It was this latter sort of success that the Keplerian theory of Mercury had achieved by May of 1661.

Claims to have observed a Mercury transit were made several times prior to the 17th century; such claims appear to have been mistaken, the observer in most cases having no doubt taken a sunspot for Mercury.[31] Kepler himself mistook a sunspot for Mercury in 1607.[32] As is well known, his prediction of a Mercury transit to occur on 7 November 1631 (N.S.) led to the first unquestioned observation of such a transit. Actually, the prediction was mistaken by some five hours in time, or 14′24″ in longitude, as Martin Hortensius, a defender of Lansberg's tables, was happy to be able to point out. [33] Lansberg's tables, Hortensius admitted, were off by 1°8′; other tables were much worse, having errors in longitude of

$4\frac{1}{2}^\circ$, 5° or more. The point worth noting is that Hortensius sees the error in Lansberg's tables as remediable, and meanwhile views Kepler's theory, closely bound up as it is with Keplerian speculations, as dubious: "Observations are to be trusted, not specious reasonings; geometrical demonstrations, not harmonic speculations."[34]

One of the most influential converts to Kepler's elliptical path and inequable motion was undoubtedly Ismael Boulliau, and Boulliau implies in 1645 that one of the important factors in his conversion was the Keplerian success with Mercury. "To bring the motions of Mercury under numerical laws, "he writes in his *Astronomia Philolaica*, "was [previously] difficult if not impossible for all [astronomers], who before Kepler used only the circular hypothesis."[35] Boulliau further points out that the inequable motion postulated by Kepler is necessary if the phenomena of Mercury are to be saved:

> ... between 1624 and 1631 I made a number of observations of Mercury, and attempted to correct the errors of the Prutenic Tables, retaining the Copernican hypothesis; but I could never get my observations to agree with the calculus, unless in the superior part of the orbit of Mercury [near aphelion] I retarded its motion, and in the inferior part [near perihelion] I speeded it up. But my attempt did not have a happy outcome, since I made this inequality of motion commensurable with the Earth's motion, while in fact it has to be referred to the motion of Mercury in its own orbit. Not yet had Kepler's tables and his *Commentaries on the Motions of Mars* fallen into my hands, and I had scruples about departing from the circular hypothesis, nor had it occurred to me that the orbit of the planet could be an ellipse: lacking which, if anyone should attempt to twist the motions of the planets into other circuits, ... he would but lose time and trouble, and revolve the stone of Sisyphus.[36]

On the other hand, it is significant that Boulliau does not announce his allegiance to the ellipse before he has hit on his own explanation for it—an explanation which is quite different from Kepler's "magnetic philosophy", and which derives the non-uniform motion on the ellipse from angularly uniform motion that shifts continuously from one to another circle in an infinite set of circles arranged to form a cone.[37]

Further empirical evidence was added in 1651 and again in 1661 by a second and third observation of a Mercury transit. These observations confirmed, Vincent Wing wrote in his *Astronomia Britannica* of 1669, that Kepler was the chief "instaurator" or restorer of astronomy:

> But this is proved especially by the planet Mercury, which on 28 October 1631, and again on 23 October 1651 and 23 April 1661 [the dates are old-style], was interposed between

> our vision and some part of the body of the Sun; on each occasion the Keplerian Tables, conforming to the Copernican hypothesis, best agreed with the truth, while the tables of Longomontanus and Argolus, conforming to the Tychonic system, contained errors of many days.[38]

In Wing's view, then, the three Mercury transits impressively confirmed the Copernican system in the form that Kepler had given it. By no means did they prove the Keplerian theory correct, or disprove the possibility that another theory, built on different foundations, could do as well. Newton, reading Wing's *Astronomia Britannica* shortly after its publication, could still be in doubt as to the exact ellipticity of the planetary path. But it is noteworthy that the alternative hypothesis he outlines on the endpapers of his copy of the *Astronomia Britannica*, while allowing for adjustment of orbital shape and motion to fit observation, nevertheless assumes an oval orbit with line of symmetry passing through the Sun.[39] A zeroth Keplerian law, the intersection of the orbital axes in the Sun, is here taken for granted; and the oval orbit, traversed under the influence of forces, is also an originally Keplerian idea.

Comparing Kepler's theory of Mercury with the corresponding Ptolemaic and Copernican theories, Owen Gingerich has shown that Kepler reduces the errors in geocentric longitude by some two orders of magnitude.[40] For a mid-17th century observer, we must not imagine that the extent and assured character of this improvement was obvious: observational error of observers like Riccioli could be several times what it had been in Tycho's observations, still under dispute were the magnitude of the corrections required for refraction and parallax, and Mercury itself was seldom observable. Meanwhile, the observed positions of Jupiter and Saturn were failing by variable amounts to accord with the Keplerian tables—the result of an inequality that first Kepler and then Jeremiah Horrox began to suspect, and that Newton with a new hypothesis as to its possible meaning is asking Flamsteed to look for in December, 1684. The empirical success of the Keplerian tables was not uniform or uniformly overwhelming, but it was sufficient to put astronomers generally onto a quite new way of analyzing their data, one that was heliocentric and heliodynamic—with orbital axis passing through the Sun and the planet varying its speed according as it approached or receded from the Sun—as the Copernican formulations were not. It is the thorough-going and indeed revolutionary character of this shift in paradigms, as it affects the inner planets, that the preceding analysis has been designed to elucidate.

NOTES

1. Philadelphia: The American Philosophical Society, 1964.
2. The claim is made for Ptolemy's theory of Mercury by Derek J. Price, *The Equatorie of the Planetis* (Cambridge: At the University Press, 1955), p. 102. Price makes a similar claim in his "Contra-Copernicus: A Critical Re-estimation of the Mathematical Planetary Theory of Ptolemy, Copernicus, and Kepler", in Marshall Clagett (ed.), *Critical Problems in the History of Science* (Madison: Univ. Wisconsin Press, 1959), p. 209, stating that Ptolemy's theory of Mercury "accorded exactly with observation". Similarly, Arthur Czwalina in "Ptolemaeus, die Bahnen der Planeten Venus und Merkur". *Centaurus 6* (1959), assumes that Ptolemy's observations of Venus and Mercury can be expected to be accurate to within about 10′ of arc. A correct view on this matter, backed by computer comparison between Tuckerman's ephemeris and ephemerides derived from the Alphonsine and Prutenic tables, is given in Owen Gingerich, "The Mercury Theory from Antiquity to Kepler", *Actes du XII[e] Congres International d'Histoire des Sciences*, Tome IIIA (Paris: Albert Blanchard, 1971), pp. 57–64.
3. Johannes Kepler, *Gesammelte Werke*, III (ed. Max Caspar, Munich: C. H. Beck, 1937), p. 109, lines 19–21. This and other volumes of the *Gesammelte Werke* will be referred to by the initials G.W.
4. G.W. I, p. 77.
5. G. W. XIV, # 190, Kepler to Magini in Bologna, pp. 174–176.
6. *Ibid.*, p. 175, lines 87–90.
7. G. W. VII, p. 434, lines 13–15.
8. J. L. E. Dreyer, *A History of Astronomy from Thales to Kepler* (2nd ed., New York: Dover, cop. 1953), p. 359.
9. Kepler did not at the time of writing the *Mysterium Cosmographicum* understand that the Copernican "equatorial epicycle" for the superior planets produces a motion equivalent to that produced by the Ptolemaic equant with bisection of the eccentricity, except for one slight difference, namely that the planet does not follow an exactly circular path but rather one that is bowed out at the sides, between aphelion and perihelion. Maestlin set him straight on this matter in a letter of 9 March 1597 (G. W. XIII, p. 111).
10. Bernard R. Goldstein, "The Arabic Version of Ptolemy's *Planetary Hypotheses*", *Transactions of the American Philosophical Society*, Vol. 57 (1967), Part IV, p. 7.
11. G. W. XIV, # 190, Kepler to Magini in Bologna, June 1601, pp. 175–176.
12. G. W. VII, p. 435, lines 41–43.
13. *Ibid.*, p. 436, lines 19–22.
14. G. W. XIV, # 203, Kepler to Maestlin in Tubingen, 10/20 December 1601, p. 204.
15. I have used the values given by J. M. A. Danby, *The Fundamentals of Celestial Mechanics* (New York: The Macmillan Company, cop. 1962), pp. 330–331.
16. Arthur Czwalina, "Ptolemaeus, die Bahnen der Planeten Venus und Merkur", *Centaurus 6* (1959), p. 17. Czwalina's ingenious investigation culminates in a comparison of the time intervals between Ptolemy's observations and those required by Kepler's second law. However, the fact that Ptolemy and Theon may have determined the *times* of maximum elongations with no more error than a week or so implies nothing with respect to the accuracy of the angles Ptolemy finds for the maximum elongations;

these angles depend on the accuracy of Ptolemy's positions for the Mean Sun and for the stars from which he measures, as well as on the accuracy of the actual measurement at the time of observation. According to Czwalina, there is a change of 10′ in Venus's elongation in the 7.7 days that follow or precede maximum elongation; I compute this same change to be more nearly $13\frac{1}{2}'$ on the average.

17. Czwalina, *op. cit.*, pp. 16–17.
18. The problem raised by the equinox and solstice observations that Ptolemy claims to have made in the *Almagest* has recently been re-examined by John Phillips Britton (*On the Quality of Solar and Lunar Observations and Parameters in Ptolemy's Almagest*, Yale University Ph. D. Dissertation, 1967). Britton concludes: "It is evident that Ptolemy's equinox observations cannot be understood as independent observations affected by an inadvertent systematic error or even as consistent observations designed to verify Hipparchus's solar parameters" (p. 42).
19. C. H. F. Peters and E. B. Knobel, *Ptolemy's Catalogue of Stars* (Carnegie Institution of Washington, Publication ≠ 86, 1915), pp. 8, 15.
20. Jean Baptiste Joseph Delambre, *Histoire de l'astronomie ancienne* (Paris, 1817), p. 333.
21. Peters and Knobel, *op. cit.*, p. 36, star ≠ 393 and p. 39, star ≠ 553.
22. G. W. XIV, pp. 175, 204.
23. G. W. XIV, p. 175.
24. G. W. XV, pp. 267–268.
25. G. W. VII, p. 435, lines 41–43.
26. In the computations of elongations from Figure V to be reported, an approximate correction has been made for the *curtatio*, that is, for the reduction to the ecliptic. The maximum correction here cannot exceed about 11′ of arc, and I would estimate that the error introduced by the approximation does not exceed 2′ or 3′.
27. Czwalina, *op. cit.*, p. 27, proposes shifting the dates of both the Ptolemaic observations listed in Table V a year back, to the 18th year of Hadrian, in order to bring the Ptolemaic errors in the intervals between observations within a limit of 4.4 days, on the assumption that the error in the observed elongation does not exceed 10′ (in the first 2.2 days before or after maximum elongation, Czwalina calculates, the elongation changes on the average by 10′). It should already be apparent that the errors in the elongations Ptolemy uses are often much larger than 10′. Comparison with the Tuckerman ephemeris shows that the shift in date would increase the error in the longitude that Ptolemy reports for Mercury by about 12° in the case of the first observation, and by about 7° in the case of the second. It is also worth noting that the shift would change the difference between Ptolemy's position for the Mean Sun and the position derived from the Tuckerman ephemeris, in the case of both observations, to about −1°23′, an increase in the absolute value of the difference by about 15′; whereas all cross checks of dates indicate that the difference at the time of Ptolemy's observations should be much closer to the smaller figure found in Table V. The proposed shift in dates does not therefore appear to be feasible.
28. Ptolemy gives the date of this observation as morning, 23–24 Mesore, 2 Antonine, which is 8 July 139. But this date is 398. 5 days after the date of the third observation in Table IV, hence according to Ptolemy's table of the Sun's regular motion, the Mean Sun should have advanced 360°+32°47′ beyond its position in that observation, which Ptolemy gives as 70°30′. Therefore the Mean Sun at the time of the present observation

should be in 103°17′. Ptolemy, however, gives its position as 100°20′. The date we have used, 20–21 Mesore, 2 Antonine = 5 July 139, brings the Mean Sun precisely to 100°20′. Cross checks with dates of other observations and the position of the Mean Sun that Ptolemy gives in each case yield the same result. As is shown in the text, however, Mercury was closer to maximum elongation on July 8 than on July 5.

29. The reciprocal of *BH* was approximated by the series:

$$1 + \frac{3e}{4} + \frac{235e^2}{64} - (2e + 4e^2)\cos\alpha - \left(e - \frac{41e^2}{32}\right)\cos 2\alpha$$
$$+ \left(\frac{e}{4} - \frac{11e^2}{32}\right)\cos 4\alpha + \frac{3e^2}{64}\cos 8\alpha - \left(\frac{e}{2} - \frac{3e^2}{8}\right)\sin\alpha\sin 2\alpha$$
$$+ \frac{11e^2}{2}\cos\alpha\cos 2\alpha - \frac{7e^2}{4}\cos\alpha\cos 4\alpha - \frac{13e^2}{32}\cos 2\alpha\cos 4\alpha$$
$$- \frac{e^2}{4}\sin\alpha\sin 4\alpha - \frac{3e^2}{8}\sin\alpha\sin 2\alpha\cos 4\alpha.$$

Here $e = 0.04749$. To obtain the sine of angle θ in Figure VIII the preceding series is multiplied by Ptolemy's value for the radius of the epicycle, namely 0.3572.

30. Let $CD = e_1$ and $DE = e_2$; then approximately

$$PR = \frac{(e_1 - e_2)^2}{2} + (e_1 + e_2)\cos\alpha - \frac{(e_1 - e_2)^2}{2}\cos 2\alpha.$$

If r_0 is the mean radius of Mercury's orbit, then θ, the angle of elongation of Mercury from the center of its orbit, or half the sum of the greatest eastern and western elongations, is given by

$$\sin\theta = \frac{r_0 - e_1\cos 2\alpha.}{PR}$$

Finally, the angle φ may be found by adding to or subtracting from α the inclination of PR to AB, given by

$$\text{slope } PR = \frac{\sin\alpha + e_2\sin 2\alpha}{e_1 + \cos\alpha + e_2\cos 2\alpha}.$$

31. Bernard R. Goldstein, "Some Medieval Reports of Venus and Mercury Transits", *Centaurus 14* (1969), p. 49 ff.
32. See Edward R. Rosen, *Kepler's Conversation with Galileo's Sidereal Messenger* (New York and London: Johnson Reprint Corp., 1965), pp. 97–99.
33. Martin Hortensius, *Dissertatio de Mercurio in sole viso et Venere invisa: instituta cum clarissimo ac doctissimo viro, D. Petro Gassendo...*, Leiden, 1633.
34. *Ibid.*, p. 68.
35. Ismael Boulliau, *Astronomia Philolaica* (Paris, 1645), p. 355.
36. *Ibid.*, pp. 355–356.
37. A description of Boulliau's theory is given in Curtis A. Wilson, "From Kepler's Laws So-called, to Universal Gravitation: Empirical Factors", *Archive for History of Exact Sciences, 6* (1970), p. 111 ff. Boulliau in his earlier *Philolai, sive dissertationis de vero systemate mundi, libri IV* (Amsterdam, 1639), while defending the Copernican system

had neither mentioned nor endorsed the elliptical path. When he states in the preface to this work that "we draw back from physical conjectures, for they do not suffice, nor is there great force in them", he is probably referring tacitly and with disapproval to the Keplerian theory.

38. Vincent Wing, *Astronomia Britannica* (London, 1669), Preface to the Candid Reader.
39. This hypothesis of Newton's is described by Derek T. Whiteside, "Newtons's Early Thoughts on Planetary Motion: A Fresh Look", *British Journal for the History of Science 2* (1964), 125.
40. See the article cited in note 2 above, and also Owen Gingerich, "Kepler's Place in Astronomy," to appear in *Vistas in Astronomy 16 (London, 1973), ed. Arthur Beer.*