Hipparchus' Empirical Basis for His Lunar Mean Motions

A historical footnote to Olaf Pedersen, *A Survey of the Almagest*, 161–64

by

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In *Almagest* IV 2 Ptolemy gives a theoretical and historical discussion of the problem of determining the moon's mean motions in longitude, anomaly and latitude. He says:

> The ancient astronomers, with good reason, tried to find some period in which the moon's motion in longitude would always be the same, on the grounds that only such a period could produce a return in anomaly. So they compared observations of lunar eclipses . . . and tried to see whether there was an interval, consisting of an integer number of months, such that, between whatever points one took that interval of months, the length in time was always the same, and so was the motion [of the moon] in longitude, [i.e.] either the same number of integer revolutions, or the same number of revolutions plus the same arc.[1]

After mentioning as a discovery of "the even more ancient astronomers" the 18-year eclipse period of 223 lunations, 239 returns in anomaly and 242 returns in latitude, which he calls the "Periodic" (miscalled "Saros" in modern times), and the triple of the latter, called the "Exeligmos", he continues as follows.

> However, Hipparchus already proved, by calculations from observations made by the Chaldaeans and in his time, that the above relationships were not accurate. For from observations he set out he shows that the smallest constant interval defining an ecliptic period in which the number of months and the amount of motion is always the same is 126007 days plus 1 equinoctial hour. In this interval he finds comprised 4267 months,

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> 4573 complete returns in anomaly, and 4612 revolutions on the ecliptic less about 7½°, which is the amount by which the sun's motion falls short of 345 revolutions (here too the revolution of sun and moon is taken with respect to the fixed stars). (Hence, dividing the above number of days by the 4267 months, he finds the mean length of the [synodic] month as approximately 29; 31, 50, 8, 20 days.) He shows, then, that the corresponding interval between two lunar eclipses is always precisely the same when they are taken over the above period [126007^d 1^h]. So it is obvious that it is a period of return in anomaly, since [from whatever eclipse it begins], it always contains the same number [4267] of months, and 4611 revolutions in longitude plus 352½°, as determined by its syzygies with the sun.

Ptolemy then remarks (1) that division of the above period by 17 produces the equivalence, in 251 months occur 269 returns in anomaly (which is not however an eclipse-period) and (2) that the above period does not contain an integer number of returns in latitude, for which Hipparchus found (allegedly also from eclipse observations) another equivalence, namely that 5458 months contain 5923 returns in latitude.

An epoch in our understanding of Greek astronomy was marked by the publication in 1900 of Kugler's *Babylonische Mondrechnung*. After quoting and paraphrasing (pp. 6–8) the passage concerning Hipparchus which I have translated above, Kugler proceeded to show that the following relationships are implicit in the cuneiform tablets which he had deciphered:

[1] In 251 synodic months occur 269 returns in anomaly (p. 21).
[2] The mean value of the synodic month is 29; 31, 50, 8, 20 days (p. 24).[2]
[3] 5458 synodic months equal 5923 draconitic months (p. 40).
[4] 1 year equals 12; 22, 8 synodic months (p. 46), and hence the length of the sidereal month agrees[3] with that derivable from the statement of Hipparchus, that the moon travels $(4612{\cdot}360-7\frac{1}{2})^\circ$ in 126007^d 1^h.

Although he could not show that the tablets from which the above relationships were extracted predate Hipparchus, Kugler rightly concluded that the priority belonged to the Babylonians (as has been amply confirmed by subsequent investigations of the cuneiform material), i.e. that Hipparchus had not, as one would infer from Ptolemy's account, derived the above relationships from observations, but simply taken them directly from a Babylonian source.

The enormous significance of this discovery for the development of Greek scientific astronomy was recognized by Franz Cumont, who disseminated Kugler's results in two articles (Cumont [1] and Cumont [2], whence Rehm in his article on Hipparchus in RE). Olaf Pedersen too, in the passage I have cited at the beginning, gives due credit to Kugler. But despite these and other mentions, Kugler's book remains to this day largely unread, and his discoveries about Hipparchus are unknown to or ignored by many of those who have written on Greek astronomy in this century.[4] Even the article by A. Aaboe on the same topic makes no reference to Kugler. However, Aaboe did not merely repeat Kugler's results, but made a significant advance in our understanding of what Hipparchus was trying to do. He showed that, starting from the Babylonian equivalence, 251 months contain 269 returns in anomaly ([1] in Kugler), Hipparchus sought a multiple of it which would be an eclipse-period, i.e. one in which, if there were a lunar eclipse at the beginning of it, there would be a good chance of there being another at the end. For this one requires (1) that there be a return in solar anomaly (i.e. that the period contain an integer number of years) and (2) that there be a return in latitude. Using as controls some value for the length of the year, e.g. 1 (sidereal) year equals 12; 22, 8 synodic months ([4] in Kugler), and the Babylonian latitude period, 5458 synodic months contain 5923 returns in latitude ([3] in Kugler), Hipparchus found that 17 was the smallest multiple which would approximately satisfy both the above conditions.[5] Multiplying the Babylonian value for the synodic month, 29; 31, 50, 8, 20^d ([2] in Kugler), by 17 times 251, he found $126007^d\ 1^h$ to the nearest hour.[6] Thus Aaboe demonstrated that the numbers attributed to Hipparchus by Ptolemy were derived by strictly arithmetical manipulation of well-attested Babylonian parameters, for the purpose of *computing* an eclipse-period.

Despite the neglect of or failure to understand Aaboe's paper by several subsequent writers on the topic,[7] one might suppose that there is nothing more that one can usefully add. However, neither Kugler, Aaboe nor any other account that I have seen gives a clear picture of what Hipparchus was really doing. The reader, comparing Ptolemy's account quoted above with the modern explanation of the Babylonian origin of all the basic parameters, might incautiously suppose that Hipparchus fraudulently tried to pass off as his own discovery, derived from observations, parameters which he had merely copied from a Babylonian source. The primary purpose of this article is to explain Hipparchus' procedure as rational and justified.

While we cannot be sure that the misleading impression which one derives from the *Almagest* of what Hipparchus did is not ultimately the fault of Hipparchus himself, my own opinion is that the fault more probably lies with Ptolemy, whose didactic purposes may have led to an "idealized" account of the way an astronomer operates, starting from observations. In any case, in order to understand the rational basis of Hipparchus' procedure, we have to read the passage in the *Almagest* following that quoted above. Ptolemy gives a long and detailed theoretical discussion of the conditions that must be satisfied, for both sun and moon, in order to deduce an "eclipse-period" from observations of eclipses. The basic principle is that one must find (at least) two pairs of eclipses which are separated by exactly the same time interval; but these have to be very carefully chosen, with certain conditions being fulfilled and certain situations being avoided, in order for the method to work.[8] Now Ptolemy does not himself establish any eclipse-period, but simply accepts (provisionally) Hipparchus' parameters (later applying corrections on the basis of the models he develops). But the point of his discussion becomes clear at the end, where he says:

> We have made the above remarks, not to disparage the preceding method of determining the periodic returns, but to show that, while it can achieve its goal if applied with due care and the appropriate kind of calculations, if any one of the conditions we set out above are omitted from consideration, even the least of them, it can fail utterly in its intended effect; and that, if one does use the proper criteria in making one's selection of the observational material, it is difficult to find corresponding [pairs of eclipse] observations which precisely fulfill all the required conditions.[9]

His discussion, then, is a critique of a real procedure involving the comparison of the intervals between two pairs of eclipses. This comparison was indeed made by Hipparchus (as is proven, if it were necessary, by the passage I quote below). We can now explain Hipparchus' procedure as follows. For his lunar theory he needed to establish the mean motions of the moon in longitude, anomaly and latitude. The best data available to him were the Babylonian parameters ([1] to [4] in Kugler).[10] But he was not content merely to accept them: he wanted to test them empirically, and so he constructed (purely arithmetically) the eclipse period of $126007^d\ 1^h$, then looked in the observational material available to him for pairs of eclipses which would confirm that this was indeed an eclipse period. The observations thus played a real role, but that role was not discovery, but confirmation.[11]

Not only can we be sure that Hipparchus used real eclipse observations to confirm the length of the eclipse-period, but we can say with a high degree of probability which eclipses he actually used. For in the passage immediately preceding that last quoted, Ptolemy says:

> That is why, as we can see, Hipparchus too used his customary extreme care in the selection of the intervals adduced for his investigation of this question: he used [two intervals], in one of which the moon started from its greatest speed and did not end at its least speed, and in the other of which it started from its least speed and did not end at its greatest speed. Furthermore he also made a correction, albeit a small one, for the sun's equation of anomaly, since the sun fell short of an integer number of revolutions by about ¼ of a sign, and this sign was different, and produced a different equation of anomaly, in each of the two intervals.

In other words, Hipparchus used the minimum of just two pairs of eclipses, in one of which the moon was "at its greatest speed", i.e. (to use the terminology of Ptolemy's and Hipparchus' lunar model) near perigee of the epicycle, and in the other "at its least speed", i.e. near apogee of the epicycle. Combining this information with the earlier statement that Hipparchus used "observations made by the Chaldaeans and in his time", we look for pairs of lunar eclipses (1) separated by an interval of 126007 days, (2) the first of which was visible in Babylon and the second in Rhodes, (3) where the second falls within the working lifetime of Hipparchus, and (4) in which the moon is either near apogee or near perigee of the epicycle.

Table 1 lists all 31 pairs of eclipses satisfying conditions (1) to (3), where the "working lifetime of Hipparchus" is interpreted very broadly to include the years from –170 to –120. The data in columns 2 to 5 are taken from Oppolzer's *Canon der Finsternisse*. The times are in all cases the time of mid-eclipse, in hours from midnight, Greenwich mean time. (To get the approximate local times in Babylon and Rhodes, add 3 hours and 2 hours respectively; precise local times in Babylon, computed from more recent elements, are given by

No.	Eclipse visible at Babylon		Eclipse visible at Rhodes		Mean anomaly
	Date	Time (GMT)	Date	Time (GMT)	
1	-514 VIII 17	2;7	-169 VIII 13	3;23	92;12
2	-512 XII 20	15;44	-167 XII 16	16;25	122;55
3	-511 VI 14	23;12	-166 VI 11	0;53	266;58
4	-511 XII 10	1;2	-166 XII 6	1;31	72;53
5	-508 X 7	16;4	-163 X 3	16;56	244;20
6	-507 IV 3	20;14	-162 III 30	21;29	52;23
7	-504 I 31	19;9	-159 I 26	20;3	227;44
8	-504 VII 27	3;47	-159 VII 23	5;1	25;6
9	-501 XI 19	21;24	-156 XI 14	21;36	2;46
10	-500 XI 7	21;3	-155 XI 3	21;28	307;42
11	-497 III 14	16;46	-152 III 9	17;45	342;22
12	-497 IX 7	21;34	-152 IX 2	22;48	137;37
13	-496 III 2	19;14	-151 II 26	20;13	288;43
14	-493 I 1	0;28	-149 XII 28	1;9	120;35
15	-490 IV 25	19;55	-145 IV 21	21;23	100;14
16	-490 X 19	0;32	-145 X 15	1;19	241;49
17	-489 IV 15	3;8	-144 IV 10	4;26	49;5
18	-489 X 8	14;41	-144 X 3	15;38	194;37
19	-486 II 11	3;28	-141 II 7	4;27	225;13
20	-485 I 31	19;3	-140 I 27	20;1	178;41
21	-483 XI 30	5;26	-138 XI 26	5;43	0;6
22	-482 XI 19	5;18	-137 XI 15	5;37	305;3
23	-479 III 24	23;41	-134 III 21	0;45	339;6
24	-479 IX 18	5;51	-134 IX 14	6;57	134;58
25	-478 III 14	2;38	-133 III 10	3;43	285;42
26	-478 IX 7	17;31	-133 IX 3	18;39	86;20
27	-475 XII 31	17;44	-130 XII 27	18;13	67;48
28	-471 X 18	23;23	-126 X 15	0;17	192;14
29	-468 VIII 17	17;37	-123 VIII 13	18;43	18;24
30	-467 II 11	3;32	-122 II 7	4;34	176;16
31	-467 VIII 6	17;46	-122 VIII 2	18;59	323;32

Table 1

Goldstine, *New and Full Moons*). Column 6 gives the mean anomaly of the moon at approximately the time of the second eclipse of each pair.[12] In order to satisfy condition (4) we look for those values where the anomaly is near 180° (greatest speed) or 0° (least speed).

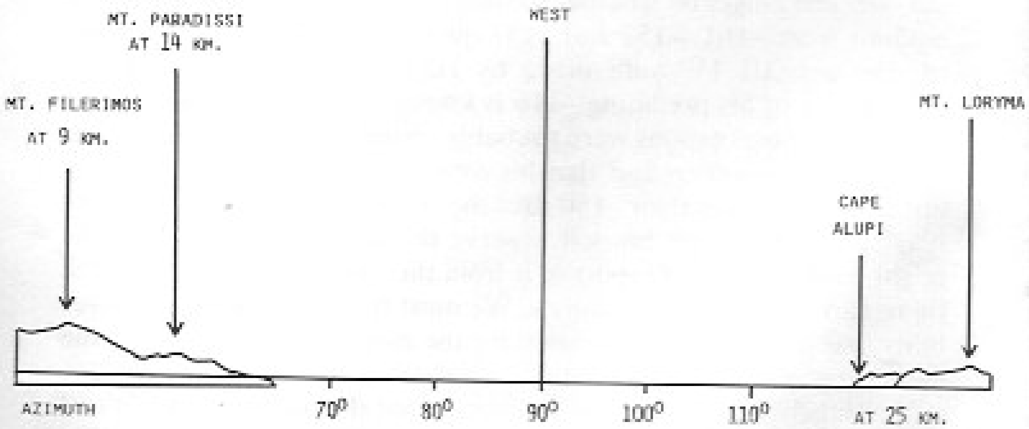
Inspection shows that for both greatest speed and least speed only two pairs of eclipses come into consideration: for greatest speed (anomaly near 180°) pair no. 20 and pair no. 30; and for least speed (anomaly near 0°) pair no. 9 and pair no. 21.[13] For the greatest speed we can immediately eliminate pair no. 30. Not only is the second eclipse, dated –122 Feb. 7, much later than the last attested observation of Hipparchus (–126 July 7),[14] but everything we know or can infer about the chronology of Hipparchus' work suggests that the basic lunar theory preceded much else. It is therefore certain that for the moon at greatest speed he used pair no. 20, the eclipse of –485 Jan. 31 (a total eclipse which we can now assume to have been observed in Mesopotamia, although it is not otherwise attested), and the eclipse of –140 Jan. 27. It is gratifying to find that the latter is referred to twice in the *Almagest* (VI 5 and VI 9). In the second passage it is explicitly stated that Hipparchus used this eclipse, and that in it he assumed the moon to have been exactly in the perigee of the epicycle.[15]

The choice for the pair at least speed is less easy. An obvious point in favor of pair no. 9 is that the first of the pair, the Babylonian eclipse of –501 Nov. 19, is attested as having been used (for an unspecified purpose) by Hipparchus.[16] But it is dubious whether Hipparchus was active as an observer as early as the date of the second eclipse of the pair, in –156. The debate on the period of Hipparchus' activity is an old one, and hinges on whether the three observations of the autumnal equinox from –161, –158 and –157 quoted by Hipparchus according to *Almagest* III 1[17] were made by Hipparchus himself (no other observation of his predating –146 is known). My own opinion is that those three observations were probably communicated to Hipparchus by an older observer, and that his own astronomical activity should not be dated earlier than –150. But there is no proof of this, and even if Hipparchus did not himself observe the lunar eclipse of –156, he might have obtained a report of it from the same source as he got the three early equinox observations. We must therefore admit the possibility that the eclipse pair he used for the moon at least speed was the earlier, no. 9.

Nevertheless, I am inclined to believe that the pair which he in fact used was the later, no. 21. Against this one may argue, first, that neither eclipse of the pair is explicitly attested in our sources, and, secondly, that it is doubtful whether the first of the pair, that of –483 Nov. 30, was visible at all at Babylon.[18] But those responsible for recording eclipses at Babylon will have been watching for it, and even if only the very beginning of the eclipse was visible, will have noted it as observed. Furthermore, there are good arguments for supposing that Hipparchus did in fact observe the second eclipse of the pair, that of −138 Nov. 26, at Rhodes.

This was a large total eclipse (18.4 digits according to P. V. Neugebauer, *Spezieller Kanon*), which began well before sunrise at Rhodes (which was shortly before 7 a.m. on the date in question). Totality began soon after 6.30 a.m., and at sunrise sun and moon were only a very short distance from exact opposition. This means that, if Hipparchus was watching the eclipse from a place where he had a clear view of both the north-west and south-east horizons, he would have seen at sunrise a strange and beautiful thing: the fully eclipsed moon was just on the north-west horizon at the same moment as the risen sun was just on the south-east horizon. (The principal reason for this paradox is refraction, which elevated each of the apparent bodies about 35′ at the horizon, and was only partially compensated by the lunar parallax.)

While we cannot know what Hipparchus' observing conditions were, we can investigate the possibilities of horizon observations in



PROFILE OF WESTERN HORIZON FROM MT. STEPHANOS ON RHODES
(AFTER FRANS BRUIN)

Figure 1

Rhodes at that time of year. For the phenomenon described to be visible, one needs a clear horizon (which means in practice a sea horizon) in both directions. Looking towards the east and northeast from Rhodes, one sees the high mountains of Lycia.[19] This means that during the summer months true sunrise could not be observed from the neighborhood of the city of Rhodes (and the situation would not be very different further south on the island). But from September to March, looking towards the southeast over open sea, one would be able to observe the sun rise over a clear horizon from any place on Rhodes where there were no local obstructions. For the other direction, I give in Fig. 1 a profile of the western horizon as seen from Mt. Stephanos (the citadel of the ancient city). This is copied from the interesting but unfortunately rather inaccessible report by Frans Bruin of his investigations on Rhodes. Bruin found a site on Mt. Stephanos which he "optimistically" (to use his own word) identified as the observatory of Hipparchus. Without necessarily following him in this, I note as pertinent to this discussion his statement that from that spot he could see three-quarters of the whole horizon, and observed both sunrise and sunset (in early September).

On the day of the eclipse in question, Nov. 26 of −138, the sun rose well south of east, at an azimuth of about 64° E for an observer in Rhodes. So the moon set almost exactly opposite, at an azimuth of about 116° W. From the considerations of the preceding paragraph and Fig. 1, we see that both would have been visible simultaneously for a suitably placed observer in Rhodes.

Now we find in the elder Pliny the following passage:

> Within the last two hundred years it has been established by the cleverness of Hipparchus that a lunar eclipse sometimes occurs at an interval of five months from the preceding one, and a solar eclipse at an interval of seven months; and that the sun can be eclipsed above the earth twice in 30 days, but that [the two eclipses] are seen by different observers; and, the most marvelous thing in this marvel, although it is agreed that the moon is darkened by the earth's shadow, [and] that this occurs sometimes on its western side and sometimes on its eastern, [he explained] the reason why, although that darkening shadow ought to be below the earth at sunrise, it has once happened that the moon was eclipsed at its setting, with both luminaries being visible above the earth.[20]

We conclude from this that in a work on eclipses Hipparchus discussed the striking phenomenon we have mentioned, and stated that it had been observed "once" (semel). It is very plausible that he was talking about his own observation of the eclipse of −138 Nov. 26 (certainly no other eclipse visible in Rhodes during his lifetime could have afforded him this spectacle).

I conclude, then, that it is probable that Hipparchus did observe the eclipse of −138. This creates a presumption (but no more) that he used the eclipse pair no. 21 as a partial basis of his confirmation of the eclipse-period. If that was in fact the pair he used, it follows that the eclipse of −483 Nov. 30 was recorded as observed in Mesopotamia (which would be of both astronomical and historical interest), and that Hipparchus' lunar theory was established later than −138. It was in any case established later than the second eclipse of the other pair used, i.e. −140 Jan. 27.

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NOTES

1. ed. Heiberg 269. All translations of passages from the *Almagest* are taken from my forthcoming English version of the whole work.
2. Kugler, who had not yet realized the advantage of expressing all Babylonian values sexagesimally, gives the equivalent fraction in hours, minutes and seconds.
3. In fact the two values do not agree precisely (*contra* Mercier, "Precession", 198), but differ by about ½ second (Kugler calculated only to the nearest second). This corresponds to the difference between the decrement of 7½° stated by Hipparchus and that of 7; 46° found by the method of Aaboe, 124.
4. Neglect of Kugler's discovery by classical scholars has been the rule, e.g. there is not a word about it in Heath's *Greek Astronomy* of 1932. (A notable exception to this rule was Fotheringham's "Indebtedness of Greek to Chaldaean Astronomy"). For examples of neglect by historians of science see notes 6 and 7 below.
5. For details of the calculation see Aaboe, 123–25, or Neugebauer, HAMA 311–12. The period falls short of 345 sidereal returns of the sun by about 7; 46° (assuming the above length for the year), which was rounded by Hipparchus to "a quarter of a sign" or 7½°. (For the use of the latter as a unit in Greek astronomy see Neugebauer [2] 250–51). The factor 17 produces, not a return in latitude, but (approximately) half a revolution beyond complete

returns, i.e. if the moon is near a node at the beginning of the period, it will be near the opposite node at the end, hence eclipses at both ends are possible.

6. This last number is of course slightly rounded. The clearest proof that the Babylonian value 29; 31, 50, 8, 20^d was what Hipparchus started from, and not (as Ptolemy's account implies) what he derived, is that division of 126007^d 1^h by 4267 produces 29; 31, 50, 8, 9 . . .^{d}. The latter value was therefore attributed to Hipparchus by Copernicus (*De revolutionibus* IV 4, cf. Aaboe, 123), and earlier still by al-Bīrūnī (*Qānūn* VII 2, II p. 730 lines 10–11). More surprisingly, after the publications of Kugler and Aaboe, it is still treated as a real value by van der Waerden, "Berichtigung" 23. The "differences" between the values of Hipparchus and Babylonian System B listed in Tabelle 4 (*ibid.* 25) are entirely illusory.
7. Like van der Waerden (see n. 6), Mercier, "Precession" p. 50, believes that "Hipparchus determined the synodic month from the 345 year eclipse cycle" (which he miscalls the "exeligmos"), even though he cites Aaboe's article. His failure to understand Aaboe's arguments is further demonstrated by his remark (*ibid.* 197 n. 1) that "of course the Babylonians must have known of" the 345 year eclipse period. In a note on the above passage in the *Almagest*, Viggo Petersen remarked that the 7½° decrement must be taken as sidereal longitude, evidently unaware that he had been anticipated by Aaboe. (His other remark, that one can derive a value for the precession of about 46.8″ *per annum* by combining Hipparchus' eclipse-period with the value for the tropical year attributed to him by Ptolemy, was also anticipated, by Biot in 1843, and by Sédillot in 1840 [Sédillot, I 11–14]. This arithmetical fact is, in my opinion, of no historical significance for anyone who has examined the evidence for the chronology and basis of Hipparchus' discovery of precession) Finally, Hartner (*Gnomon* 1972, 534–35), demonstrates the equivalence of the Babylonian and Hipparchan data with no reference to either Kugler or Aaboe, and adding nothing to the latter's discussion.
8. Heiberg 272–76. The conditions are well explained and illustrated by Neugebauer, HAMA 71–73 with Figs. 59–62.
9. Heiberg 276–77.
10. We know nothing of the vehicle or even the form in which these were transmitted from Mesopotamia, but it seems vastly more probable that they were given explicitly by someone with personal knowledge of Babylonian astronomy than that they were extracted by any Greek from the kind of ephemerides known to Kugler.
11. In a forthcoming article in *Archive for History of Exact Sciences*, "Hipparchus' Determination of the Length of the Tropical Year and the Rate of Precession", N. Swerdlow makes the interesting suggestion that Hipparchus derived his length for the tropical year, 365¼ minus about 1/300 day, by arithmetically combining the Babylonian value for the mean synodic month and the Callippic 76-year cycle, and then attempted to confirm it by e.g. comparison of solstice observations. This seems very plausible, and if true is so closely parallel to his procedure in the lunar theory that one must suppose that Hipparchus' success in the latter encouraged him to apply the same method in the solar theory.
12. I have computed these from the Almagest tables, for a time 2 hours later than the time of mid-eclipse as computed by Oppolzer (since Ptolemy's tables have an epoch of noon Alexandria). We cannot reconstruct exactly the positions in mean anomaly of the moon as given by Hipparchus' tables, since, although we know what value he used for the mean motion, we do not know what his epoch position was. The only passage which has been thought to give explicit evidence about this is *Almagest* V 3, Heiberg 363, 19, where

Hipparchus is quoted as saying that the speed ("dromos") is "241". This has been interpreted to mean that the anomaly is 241°, and the text consequently emended, since this is an impossibly large difference from Ptolemy's 257; 47°. However, as I shall show elsewhere, Hipparchus meant that the moon was in the 241st day of the anomaly period of 248 days well known from Babylonian, Greek and Indian astronomy. This agrees with Ptolemy's calculation, without providing a precise position. But the passage referred to in n. 15 shows that Hipparchus' epoch in mean anomaly was no more than about 2° different from Ptolemy's. Hence for our present purposes Ptolemy's values are sufficiently close.

13. Note that in each case the matching pairs are 18 years (a "Saros") apart.
14. All dated observations in the *Almagest* are conveniently listed in Appendix A of Olaf Pedersen's *Survey*.
15. Heiberg 525–26. I shall deal with this passage elsewhere, in a discussion of Hipparchus' determination of the mean motion in latitude.
16. *Almagest* IV 9, Heiberg 332. Both Aaboe (p. 125) and Hartner (p. 535 n. 1) noticed that this eclipse has a corresponding one 126007 days later and suggest that Hipparchus may have cited this pair in connection with that period. Earlier Rome, *Commentaire de Théon* 991–994 n. (2), had chosen (from a list similar to my Table 1) the pair –485 I 31 and –140 I 27 as "the example" which he guessed Hipparchus to have used. But none of them explains that Hipparchus' method requires *two* pairs of eclipses, or notes the crucial element of the lunar anomaly. If one's criterion is merely whether the eclipses in question are attested as having been observed, a far better candidate would be pair no. 15, both of which are reported in the *Almagest*, that of –490 Apr. 25 at IV 9, Heiberg 329, and that of –145 Apr. 21 at III 1, Heiberg 199, where it is said to have been used by Hipparchus to determine the longitude of Spica. But at these eclipses the moon was near mean speed, which rules them out of our investigation.
17. Heiberg 195. For some of the literature see Neugebauer, HAMA 275–76. Kugler, 50–51, argues against attributing these observations to Hipparchus.
18. P. V. Neugebauer, *Spezieller Kanon*, marks the time of eclipse-beginning at Babylon, 7.2ʰ, with a question-mark to indicate that it may have begun after sunrise. By calculation from the tables in P. V. Neugebauer, *Astronomische Chronologie*, I find the time of eclipse-beginning at Babylon as about 6; 55 a.m., and from P. V. Neugebauer, *Tafeln* III, the time of sunrise at Babylon on that date as about the same. The eclipse was a small one (about 2½ digits). Clearly, while the later phases were certainly invisible at Babylon, we cannot press calculations from modern elements so far as to affirm confidently that the beginning was or was not visible there.
19. This view is well illustrated by Plate 5 in Newton's *Travels and Discoveries in the Levant*, Vol. I, where one is looking more or less due east from the citadel of Rhodes across the harbor: the mountains block the whole horizon to the east and northeast, but to the south-east is open sea.
20. Pliny, NH II 56 (Budé p. 25). One need hardly say that Pliny's account, as usual, is both confused and obscure. But the fact of the observation, and its mention by Hipparchus, are certain. There is a discussion of the phenomenon by Cleomedes, II 6 (Ziegler 218 ff.), but without any details; indeed at one point (p. 222) he denies that such an eclipse has ever been observed.