Computing Astronomical Rising Signs for Any Latitude Based on Ptolemy's *Almagest*

Jon Chesey

Introduction

Ptolemy's *Mathematike Syntaxis*, more commonly known as the *Almagest*, is a collected overview of Greek astronomy. Compiled into thirteen books, Ptolemy laid out the mathematical foundations for his view of the geocentric universe, allowing followers of the book to make mathematically based predictions of things like the length of days and the position of objects in the sky.

While the *Almagest* does not make direct reference to astrological principles, astrology existed well before Ptolemy's time and long after and Ptolemy wrote a separate treatise on the subject, the *Tetrabiblos*. Even astronomers such as Kepler (1571-1630) garnered favor with patrons by casting horoscopes. The most well-known component of the astrological horoscope is the sun sign, which denotes the zodiacal constellation the sun was in at the moment of birth. This value is easy to determine as it is tied solely to the date and repeats yearly.

A more difficult value to determine is that of the rising sign which is the zodiacal constellation on the eastern horizon at birth. This is determined not only by date, but also latitude and time of day. Ptolemy lays out a method for determining this value in book II chapter 9 (II.9) making use of a previously derived table of rising times of the ecliptic (II.8). However, this table is only for a selected 11 latitudes. While using these tables to derive the rising sign is relatively simple, accomplished astronomers could offer a level of expertise that allowed them to reproduce the mathematical derivations Ptolemy used to produce the table of rising times, for whatever location may be necessary, promising more accurate horoscopes. But because the *Almagest* was not intended to be a work specifically for the purposes of astrology, these derivations are not laid out in a concise format.

Thus, after introducing the methodology to calculate the rising sign, the purpose of this paper is to distill the method introducing only those concepts and equations which are necessary to complete methodology and excluding their derivations. This will be done through use of an example.

However, Ptolemy uses several methodologies which will not be familiar to most readers. As such, I have collected explanations of them in the **Appendix A**. Readers are advised to familiarize themselves with this material before approaching the main text.

Example setup

The parameters that I will use for this demonstration is a date of Oct 3[1], a time of 2.5 seasonal hours after sunrise, in the Barony of Three Rivers which has a latitude of 38.63º N[2]. This latitude is between the latitudes Ptolemy uses for Rhodes, Greece (36º N) and Hellespont[3] (40;56º N) which

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[1] My birthday.

[2] This latitude is a modern value. However, Ptolemy claimed accuracy in latitude to 1 minute of arc which is a similar order of magnitude.

[3] Hellespont is the ancient name of the modern-day Dardanelles which is a natural waterway in northwestern Turkey that forms the continental boundary between Europe and Asia.

will allow for the rising time tables to be used for these locations on the same date and time, ensuring that the calculated value falls between the two.

Ptolemy's methodology

Ptolemy describes the steps necessary to calculate the rising sign in II.9. There, he says[4]:

> *[G]iven any date and time whatever, expressed in seasonal hours, on that date, we can find, first, the degree of the ecliptic rising at that moment. We do this by multiplying the number of hours, counted from sunrise by day, and from sunset by night, by the relevant length of the [seasonal] hour in time-degrees. We add this product to the rising-time at the latitude in question of the sun's degree by day (or the degree opposite the sun by night): the degree [of the ecliptic] with rising-time corresponding to the total will be rising at that moment.*

While Ptolemy's explanation is complicated, the concept is quite simple. We first find the time of sunrise/sunset (in time-degrees), and then add on the additional rising time (also in time-degrees) of the segment of time to the specified hour.

Before beginning a more complete explanation suitable for any latitude, I will first demonstrate this method for the two latitudes in the rising-time table above and below the latitude taken for the example problem, beginning with Rhodes.

Rhodes

First, we must determine the position of the sun on the date in question. I have produced a table to assist with this in **Appendix B**. From there, we can see that on Oct 3, the sun is 193.32º along the ecliptic from the vernal equinox[5], placing it 13.32º (44.38%) into Libra[6].

Next, we use this to determine the cumulative rising time of that arc from the rising time table for Rhodes which I have placed in **Appendix C**. However, the table does not display the necessary value and as such, some extrapolation will be necessary.

The rising time for Libra spans from 180;0º to 216;28º time-degrees, which converted to decimal is 180.0º and 216.47º time-degrees respectively. The difference is 36.47º. Taking 44.38% of that we determine that the sun is 16.19º past the beginning of Libra at 180º. Thus, $180º + 16.19º = 196.19º$. This is the rising time, in equinoctial time-degrees, for sunrise on the given date at that latitude, since the vernal equinox last passed the horizon.

Ptolemy's method says that we will need to add to this, the length of the seasonal hours times the number of seasonal hours after sunrise. However, this means we need to determine the length (in time-degrees) of a seasonal hour. A method for doing so is described in II.9 and requires finding the rising time for sunset on the given day by much the same method as above. However, instead of using the location of the sun, we will use the location from the rising times table that is opposite it. In

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[4] Ptolemy, and G. J. Toomer. *Ptolemy's Almagest*. Princeton, N.J: Princeton University Press, 1998

[5] This is generally known as the sun's right ascension and is generally denoted by a lowercase alpha, α. In addition, the point is no longer actually the location of the vernal equinox. Due to precession of the equinoxes, the actual equinox is located in Pisces. Instead, this point is more commonly known as the "first point in Aries." However, to remain consistent with the source material, I will continue calling it the vernal equinox.

[6] Ptolemy takes the width of each zodialogical sign to be 30º. Thus, $\frac{13.32}{30} = 44.38\%$.

other words, since Aries is located opposite Libra on the zodiac, we need to find the rising-time of the point 44.38% the way into Aries. Since Aries has rising times from 0 to 19;12º (19.2º converted to decimal) at Rhodes. Thus, 44.38% the way between these two values is 8.53º.

These two values can be used to determine the length of the day or night. To determine the length of the day, we'd go from 196.19º to 360º and then add on the additional 8.53º to get to sunset. Stating that mathematically, 360º -196.19º +8.53º = 172.34º. Since a seasonal hour is defined as $\frac{1}{12}$ of the day, this indicates the length of a seasonal hour is $\frac{172.34º}{12} = 14.36º$.

But this is the length of one seasonal hour and the example we're taking is for 2.5 hours after sunset. Thus, we multiply:

$$14.36º \times 2.5 = 35.90º$$

which we then add to the previously determined rising-time for sunrise:

$$196.19º + 35.90º = 232.09º$$

We can then look this up on the same rising-time table to determine which constellation this is and thus learn that it lies in Scorpio. That is to say, Scorpio is on the horizon 2.5 hours after sunrise on Oct 3, in Rhodes.

More specifically, we note that Scorpio begins at 216;28º (216.47º) and ends at 253;30º (253.50º) for a difference of 37.03º. The rising point is 232.09º which is 15.81º past the first point in Scorpio or 42.70% of the way into Scorpio.

Hellespont

As another example, I will repeat this procedure for Hellespont. As the date in question is still the same, the sun is again 44.38% of the way through Libra. However, at Hellespont, Libra ranges from 180.0º to 218.13º for a difference of 38.13º. Again, we take 44.38% of that to time of sunrise being 16.92º into Libra or a total rising time of 196.92º.

We next find the time of the sunset which is again in Aries. In Hellespont the total rising time of this sign is 17;32º (17.53º) and 44.38% of that is 7.78º. From this we find the length of the day by going from the previous rising time (196.92º) to 360º and adding in the additional time until sunset. Written mathematically, 360º - 196.92º + 7.78º = 170.86º. This again is the length of the day, so one daytime seasonal hour is $\frac{1}{12}$ of this, which is 14.24º.

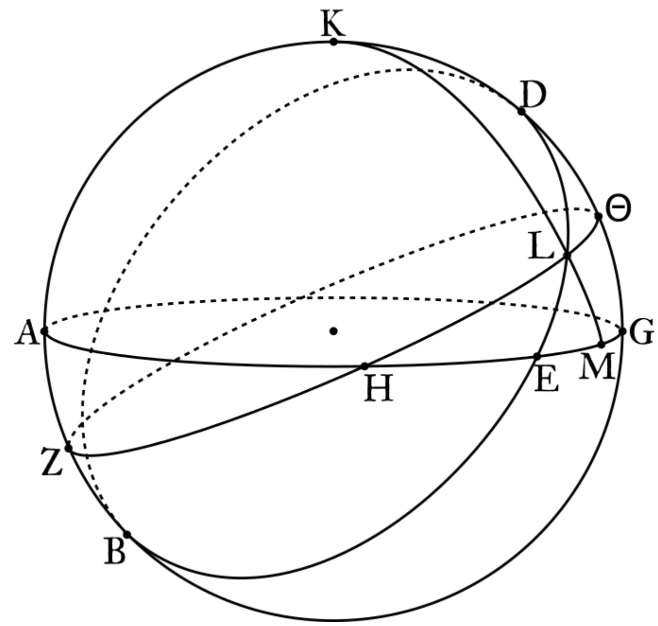
Since our time is 2.5 hours after sunrise, we take the time of sunrise from above (196.92º) and add 2.5 seasonal daytime hours to it $196.92º + 2.5(14.24º) = 232.52º$. Again, this position is looked up in the rising time table to determine that, even at this latitude, Scorpio was the sign on the horizon at that time.

As with before, we can determine the precise percent this is into Scorpio by noting that, in Hellespont, Scorpio begins at 218;8º (218.13º) and ends at 256;37º (256.17º) for a full rising time of 38.49º. The rising point at the time in the example was 13.72º past the first point or 35.65% of the way through.

Generalization

We have now demonstrated Ptolemy's method for determining the rising sign given a date, time, and location, by using the rising-time tables in II.8. However, the purpose of this paper is to distill the procedure by which Ptolemy generated his tables so we can do so for any given latitude. Ptolemy's derivation of his tables came in II.7 although, as we shall see, he made use of many previous chapters. In this chapter, Ptolemy begins by deriving a basic formula which will work for any right ascension up to 80º from the vernal equinox[7]. However, the formula is computationally challenging to do by hand, so Ptolemy goes on to derive a second method that is easier to do without computational aides[8]. But since we can make use of calculators, the first method is the only one we will explore in this paper. The objective, using this method, will be to determine the rising times, and thus cumulative rising times, for each 10º arc at the latitude in question for the full 360º of the ecliptic. Essentially, we will be adding another column to Ptolemy's table for our latitude.

To do so, we will begin with the diagram at right[9]. While the figure appears quite complicated, it is four great circles and an arc of another one.



- Great circle AEG is the celestial equator.
- Great circle ZHΘ is the ecliptic.
- Great circle BED is the horizon for the latitude in question.
- The apparent perimeter, great circle ABG is the meridian for the latitude in question.
- Arc KM is drawn in as this provides another arc of a great circle to complete the Menelaus' configuration we will explore shortly.

From this setup, we should also note a few points of interest.

- Point H is the vernal equinox.
- Point K is the north celestial pole.
- Point E is the point of the celestial equator on the horizon for the given latitude.
- Point L is the point of the ecliptic on the horizon where the sun would be located when it is rising.
- Point D is the point at the intersection of the horizon and meridian closest to K, which means it is directly north on the horizon as this is the point on the horizon closest to the north celestial pole.
- Point E is the intersection of the horizon and the celestial equator which means it is due east due to its relative position to point D.

Ultimately, our goal will be to determine the rising time, in time-degrees, of *arc* HL as that arc is a section of the ecliptic which we will be breaking into 10º segments as Ptolemy did. However, we will get at it indirectly. Given that point E is east on the horizon, this is the area of the horizon circle

[7] When the right ascension = 90º the configuration that makes the equation work collapses to a series of overlapping lines and it no longer works.

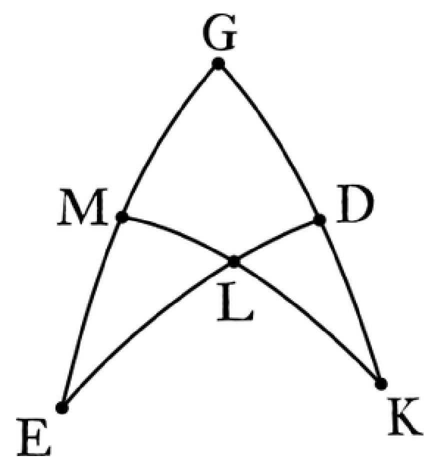
[8] In fact, Ptolemy derives a *third* method that he uses to derive the 10º intervals, but again, there is no need to discuss it here as we have better methods available to us.

[9] See: http://jonvoisey.net/blog/2018/10/almagest-book-ii-calculation-of-rising-times-at-sphaera-obliqua/ for a more thorough explanation and an example worked out for Rhodes.

that will have objects rising. Let us consider points H, L, and E. As the celestial sphere turns, point H will rise first followed by points L and E being on the horizon at the same time. This means that the rising times for *arc* HL and *arc* HE will be equal. Since *arc* HE is easier to determine, our objective here will be to determine that value. However, we will again need to make use of an intermediate step.

Determining arc EM

To begin, Ptolemy makes use of a Menelaus configuration. Pulled off the sphere and flipped horizontally it is shown at right. Using Menelaus' Theorem, we can state:



$$\frac{Crd\ arc\ 2DK}{Crd\ arc\ 2DG} = \frac{Crd\ arc\ 2KL}{Crd\ arc\ 2LM} \times \frac{Crd\ arc\ 2EM}{Crd\ arc\ 2EG}$$

Obviously, this equation does not contain *arc EH* which we are searching for, but it does contain *arc EM* which can be subtracted from *arc HM* to determine *arc EH*. Thus, we will first concern ourselves with finding *arc EM*. To do so, requires us to solve the above equation by knowing the value of the other five variables to solve for the remaining one.

First, we can note that *arc DK* is the angle of the north celestial pole above the horizon. As this is the same as the latitude, we can state that *arc DK* = 38.63º. However, we note that Menelaus' Theorem does not call for the arc; it asks for the *Crd arc* of twice the angle. Thus, twice the angle is 77.26º. The *Crd arc* of this can be looked up in I.11 or achieved more easily using modern methods via the formula I have listed in Appendix A, $Crd\ arc\ 2\theta = 120 \sin\theta$ [10], which will be my preferred method in this paper. Regardless of method, *Crd arc 2DK* = 74.91.

Next, we can determine *arc DG* by noting that *arc KG* is the angle from the north celestial pole to the celestial equator. As this is always 90º, *arc DG* is 90º - *arc DK* which was the latitude. Thus, *arc DG* = 90º - 38.63º = 51.37º and *Crd arc 2DG* = 93.74.

Arc LM is the arc between the celestial and equator when *arc HL* = 10º to begin (although we will be repeating this calculation for each 10º). In I.15, Ptolemy derived a table that can be used to determine this value (see: **Appendix D**). For the first 10º arc of the ecliptic, this gives a value of 4;1;38º or 4.03º. This gives *Crd arc 2LM* = 8.43.

Next, we can find *arc KL* by noting *arc KM* is again from the north celestial pole to a point on the celestial equator, which again means that *arc KM* = 90º. Thus, *arc KL* = 90º - *arc LM* which we just found. Therefore, *arc KL* = 85.97º and *Crd arc 2KL* = 119.70.

Lastly, *arc EG* is 90º because point E is due east on the horizon and point G is on the meridian. Since the meridian is always 90º in altitude from a point due east or west, we can state *arc EG* = 90º and *Crd arc* 2EG = *120*.

[10] Note that this modern method uses the *sin* of the arc directly. Applying the chord tables requires doubling the arc.

This gives us all the necessary values to now determine *Crd arc EM* from which we will be able to determine arc EM.

$$\frac{74.91}{93.74} = \frac{119.70}{8.43} \times \frac{Crd\ arc\ 2EM}{120}$$

Solving this for *Crd arc 2EM*:

$$Crd\ arc\ 2EM = 120 \times \frac{74.91}{93.74} \times \frac{8.43}{119.70} = 6.75$$

From this, we can determine *arc EM* = 3.23º.

Determination of arc HM

To do so, let us first consider arc KM. This arc connects the north celestial pole to the celestial equator and thus, forms a right angle with the celestial equator. If we consider arc KM to be a segment of the horizon, the celestial equator appears perpendicular to the horizon only on the terrestrial equator, a place Ptolemy refers to as *sphaera recta*[11] which I will adopt in order to avoid confusion between the celestial and terrestrial equators. At *sphaera recta*, as the sky turned, point H would rise first, and then points L and M would both be on the horizon simultaneously since they are both on arc KM which was the horizon at *sphaera recta*. Thus, at *sphaera recta*, the rising time of arc LM and arc KM are equal.

This topic was explored in I.16[12]. There, Ptolemy makes use of another Menelaus configuration in the diagram to state:

$$\frac{Crd\ arc\ 2K\Theta}{Crd\ arc\ 2\Theta G} = \frac{Crd\ arc\ 2KL}{Crd\ arc\ 2LM} \times \frac{Crd\ arc\ 2HM}{Crd\ arc\ 2HG}$$

As with before, we must have the values for the other five variables, so we may solve for *Crd arc* 2HM. Fortunately, only the terms on the left side of the equation and the one we're solving for are new. The remaining 3 were also in the previous Menelaus configuration. Thus, we need only address the terms on the left side of the equation.

First, *arc* ΘG is the angle between the celestial equator and the ecliptic[13]. Ptolemy uses a value of $\frac{11}{83}$ of 360º, for *arc* 2ΘG. Thus, *Crd arc* 2ΘG = 48.54.

Next, *arc* KΘ is from the pole to the ecliptic. Since *arc* KG is from the pole to the equator, which is 90º, we can subtract arc ΘG from 90º to determine *arc* KΘ = 90º - 23.86º = 66.14º. Therefore, *Crd arc* 2KΘ = 109.74.

Plugging these into the equations with the values found previously we get:

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[11] Lit. "The upright sphere"

[12] Ptolemy's labeling of the points was different in that section. In order to provide continuity within this paper, I have elected to remain within the context of the initial diagram.

[13] This value is referred to in Appendix A as an example of sexagesimal notation.

$$\frac{109.74}{48.54} = \frac{119.48}{8.46} \times \frac{Crd\ arc\ 2HM}{120}$$

Solving this for *Crd arc* 2HM gives us:

$$Crd\ arc\ 2HM = 120 \times \frac{109.74}{48.54} \times \frac{8.46}{119.48} = 19.21$$

Solving we get *Crd arc* 2HM = 19.10 which in turn gives us *arc* HM = 9.16º.

Determining arc EH

With valves for *arc* HM and *arc* EM in hand, we can subtract them to state

$$arc\ HM - arc\ EM = arc\ E$$

$$9.16º - 3.23º\ =\ 5.93º$$

Remaining intervals to 80º

At this point, we have derived the first line for a new column for the rising-time tables which was 10º of the ecliptic, extending from the vernal equinox to 10º into Aires. To complete the tables, we will need to repeat the above calculations 7 more times to extend this to 80º. Fortunately, very little will change between calculations, so let us now explore which values will remain consistent and which we will need to adjust. To do so, I have summarized the arcs into the table below which shows what variables the arcs will be dependent upon.

Arc Length	**Latitude**	**Neither**
arc LM = from table I.15	arc DK = latitude	*arc* EG = 90º
arc KL = 90º – *arc* LM	arc DG = 90º – latitude	*arc* ΘG = 23.86º
		arc KΘ = 66.14º

As we can see from the table above, there are only two variables which will differ as we recompute for changing lengths of arcs of the ecliptic: *arc* LM and *arc* KL. If we were to select a different location, this would change the two arcs associated with latitude as well. But since we will continue calculating arcs for the Barony of Three Rivers, these too will remain constant.

With that in mind, we can more quickly move forward with a calculation for the rising-time of a 20º arc of the ecliptic. We first determine *Crd arc* 2EM. Here, we look up a 20º arc between the equator and ecliptic from table I.15 to get *arc* LM = 7;57,3º = 7.95º and *arc* KL = 90º - 7.95º = 82.05º. Converting these to the Crd arcs, we get *Crd arc* 2LM = 16.60 and *Crd arc* 2KL = 118.85. Plugging those into the equation we get:

$$\frac{74.91}{93.74} = \frac{118.85}{16.60} \times \frac{Crd\ arc\ 2EM}{120}$$

$$Crd\ arc\ 2EM = 13.93$$

$$arc\ EM = 6.41º$$

Doing the same to get *arc* HM:

$$\frac{109.74}{48.53} = \frac{118.85}{16.60} \times \frac{Crd\ arc\ 2HM}{120}$$

$$Crd\ arc\ 2HM = 37.90$$

$$arc\ HM = 18.41º$$

Taking the difference:

$$18.41º - 6.41º = 12.00º$$

It should be restated that this value is the time it would take a 20º arc of the ecliptic to rise at the given latitude. If we wish to know how long it would take for the second 10º arc to rise, we must subtract the time it would take the first 10º arc to rise from this:

$$12.00º - 5.93º = 6.07º$$

Repeating these calculations through arcs of the ecliptic up to 80º and we generate the following table:

Sign	Total arc of the Ecliptic	º	Acc
Aries	10	5.93	5.93
	20	6.07	12.00
	30	6.33	18.33
Taurus	40	6.74	25.07
	50	7.29	32.36
	60	7.97	40.33
Gemini	70	8.77	49.10
	80	9.62	58.72
	90		

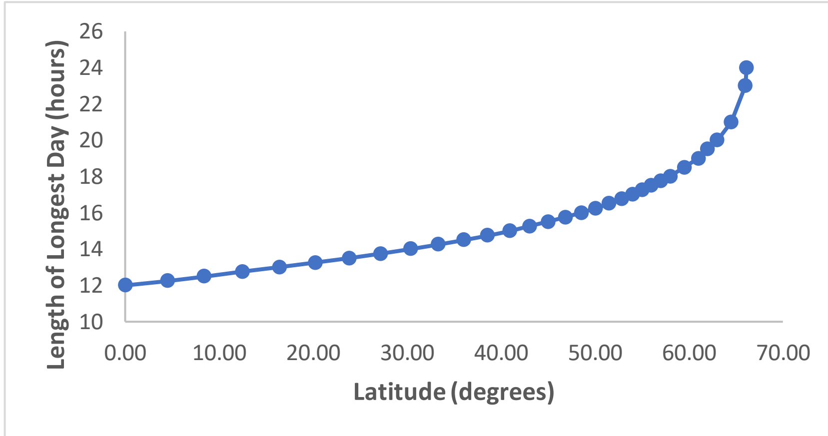
Completing the 90º arc

As noted previously, the application of Menelaus' Theorem fails when the arc of the ecliptic is 90º as there is no longer a Menelaus configuration when the lines overlap. Therefore, Ptolemy makes use of an alternative method to determine the accumulated rising time for the entire first quadrant. This method requires knowing the length of the longest day (i.e. the length of the day on the solstice). Ptolemy introduces knowing this quantity for different latitudes[14] as a one of his seven pieces of knowledge that he deems important in II.1. However, exactly how this quantity is determined is never explained. Instead, Ptolemy simply provides the values as givens for 33 different latitudes in II.6[15] which I have listed in **Appendix F**. As such, before we proceed with his methodology, we should briefly explore how we might extrapolate the length of the day from the data provided.

[14] More specifically, how it relates the seasonal and equinoctial hours.

[15] In total, Ptolemy actually covers 39 latitudes but 6 of them are above the arctic circle where during the summers the sun does not set which makes the length of the longest day undefined.

Here, I have represented the length of the longest days at the various latitudes Ptolemy listed in II.6 graphically. In that section, Ptolemy began by giving the latitude for every ¼ hour increase in the length of the summer solstice day. But it can be readily noted that the difference between such latitudes quickly diminishes which results in the points getting closer together quickly. Ptolemy shifts from ¼ hour intervals to ½ hour intervals at 18 hours, and then to full hour intervals after 20 which is why the points spread back out towards the end.



One of the things we can see from the graph is that where the points are spread well apart, the trend line is much flatter allowing for a good linear approximation between points. When the slope starts changing more rapidly, the points become closer together which means a linear approximation will still be function reasonably well[16]. Thus, we can feel justified in using the same approach we did previously in taking the proportions between points.

In our example, we have a latitude of 38.63º which falls between the values of 38.58º and 40.93º listed in the table. The difference of those two values is 2.35 and 38.63º is .05º degrees past the initial point or $\frac{.05}{2.35} = 2.13\%$ into the interval. If we apply that same 2.13% to the 0.25 hour interval we determine that this is 0.005 hours (~3 minutes) later than the 14.75 hour day. Rounding, we can therefore state that the length of the longest day for our latitude would be 14.76 hours[17].

So how is this used?[18] Ptolemy first takes the length of the *shortest* day (the winter solstice) which is 24 hours minus the length of the longest day and would be 9.24 hours in our example. The reason for this is that on the winter solstice, the sun, as it rises, would be at the first point in Capricorn. When it sets, the last point in Gemini would be just rising on the horizon. These are important because Capricorn and Gemini are diametrically opposite on the ecliptic with Aries rising half-way between the two.

From there, we can exploit one of two important symmetries Ptolemy explores at the beginning of II.7. Namely, that any two arcs equidistant from the same equinox rise in the same amount of time. This means that, since the first point in Aries is an equinox, if we know the rising

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[16] It should be noted that there is not a simple equation that represents this line as well as a series of linear approximations will.

[17] If we look up the length of the day online, we find that it is actually 15 hours, 2 minutes. The difference in these times is largely due to how the length of the day is defined. Here, Ptolemy is treating the sun as a single point that is either above or below the horizon, whereas in reality, the sunrise begins when the limb of the sun first touches the horizon, and sunset occurs when the opposite side of the disc sets. This extra duration caused by the angular size of the sun accounts for the majority of the difference.

[18] I'm skipping much of the reasoning here as it was quite long. Those interested should see: http://jonvoisey.net/blog/2018/11/almagest-book-ii-calculation-of-rising-times-at-sphaera-obliqua-for-remaining-arcs/

time for Capricorn to Gemini, we can divide it in half to find the rising time from Aries to Gemini which is the 90º arc we're seeking.

Ptolemy describes how to find the rising time as part of a longer derivation in II.2, but his reasoning is quite muddled. A simpler solution is to note that we're looking for the proportion of 360º that 9.24 hours is to a day. In other words:

$$\frac{9.24}{24} \times 360 = 138.6^{\circ}$$

Thus, the full 180º arc from Capricorn to Gemini rises in 138.6º, and the arc from Aries to Gemini rises in half that, 69.3º. This can now be added to our table and the difference from the previous cumulative rising time can be added.

Sign	Total arc of the Ecliptic	º	Acc
Aries	10	5.93	5.93
	20	6.07	12.00
	30	6.33	18.33
Taurus	40	6.74	25.07
	50	7.29	32.36
	60	7.97	40.33
Gemini	70	8.77	49.10
	80	9.62	58.72
	90	10.58	69.30

Completing the table using symmetries

At this point, we no longer require complicated calculations. First off, we can quickly complete another quarter of the table for the individual arcs by using the previously stated symmetry: arcs equidistant from the same equinox will rise in the same amount of time. Since our current table has Aries, the zero point of which is the vernal equinox, what this effectively means is that the last 10º arc of Pisces will rise in the same amount of time as the first 10º arc of Aries, the second to last 10º arc of Pisces will rise in the same amount of time as the second 10º arc of Aries, etc….

Adding this to the table to highlight the symmetry:

Sign	Total arc of the Ecliptic	º	Acc
Aries	10	5.93	5.93
	20	6.07	12.00
	30	6.33	18.33
Taurus	40	6.74	25.07
	50	7.29	32.36
	60	7.97	40.33
Gemini	70	8.77	49.10
	80	9.62	58.72
	90	10.58	69.3
. . .			
Capricorn	280	10.58	
	290	9.62	
	300	8.77	
Aquarius	310	7.97	
	320	7.29	
	330	6.74	
Pisces	340	6.33	
	350	6.07	
	360	5.93	

The second symmetry relates arcs opposite the same solstice. As Ptolemy states it,

> *If two arcs of the ecliptic are equal and are equidistant from the same solstice, the sum of the two arcs of the equator which rise with them is equal to the sum of the rising-times [of the same two arcs of the ecliptic] at sphaera recta.*

Let us use the first 10º arc of Aries as an example. Here, this arc is equally distant from the solstices as the last 10º arc in Virgo, whose rising time is currently unknown. The symmetry states that the sum of these two rising times will be equal to the sum of the rising time of the same two arcs at *sphaera recta*. Fortunately, at *sphaera recta* yet another symmetry exists such that arcs equidistant from the same solstice are equal, which is to say the time in which the first 10º of Aries will rise is the same as the amount of time it will take the last 10º of Virgo to rise. That value can be looked up in I.16, which I have reproduced as a table in **Appendix E** and is 9;10º = 9.17º. Thus, we can state:

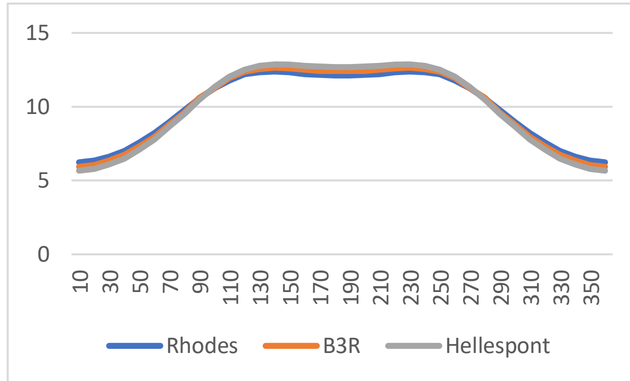
$$Rising\ time\ of\ last\ 10º\ of\ Virgo + 5.93º = 2(9.17º)$$

$$Rising\ time\ of\ last\ 10º\ of\ Virgo = 18.34º - 5.93º = 12.41º$$

This procedure can be repeated for each of the 10º arcs in the first 90º and then the other symmetry applied to complete the rising times for each arc. Lastly, we can continue adding the rising times for each 10º arc to complete the table.

Sign	Total arc of the Ecliptic	º	Acc
Aries	10	5.93	5.93
	20	6.07	12.00
	30	6.33	18.33
Taurus	40	6.74	25.07
	50	7.29	32.36
	60	7.97	40.33
Gemini	70	8.77	49.10
	80	9.62	58.72
	90	10.58	69.30
Cancer	100	11.25	80.55
	110	11.95	92.50
	120	12.36	104.86
Leo	130	12.56	117.42
	140	12.65	130.07
	150	12.59	142.66
Virgo	160	12.50	155.17
	170	12.43	167.60
	180	12.40	180.00
Libra	190	12.40	192.40
	200	12.43	204.83
	210	12.50	217.34
Scorpius	220	12.59	229.93
	230	12.65	242.58
	240	12.56	255.14
Sagittarius	250	12.36	267.50
	260	11.95	279.45
	270	11.25	290.70
Capricorn	280	10.58	301.28
	290	9.62	310.90
	300	8.77	319.67
Aquarius	310	7.97	327.64
	320	7.29	334.93
	330	6.74	341.67
Pisces	340	6.33	348.00
	350	6.07	354.07
	360	5.93	360.00

Before continuing on, a quick reality check can be applied. Given that we have similar tables for both Rhodes and Hellespont, we can graph the rising times for each 10º arc to verify that B3R's rising times fall correctly between them.



Here, we can clearly see that this is the case and the curve generated does indeed match the expected shape confirming that we have followed the methodology correctly. All that remains is to use the tables as we did for the examples to calculate the final rising sign.

Using the table to calculate rising sign

With the complete table, we can now apply the previously described method to determine the rising time. We begin by recalling that the sun was 44.38% of the way through Libra which gives a rising time to that point of 196.57º.

We next determine the rising time of the point opposite the position of the sun which was 44.38% of the way through Aires for a rising time of 8.13º. Taking difference of 360º and the sunrise time gives 163.43º, and adding the additional 8.13º to get the full length of the day gives us a total day length in equinoctial time-degrees of 171.56º. Dividing that by 12 gives us a seasonal daytime hour of 14.30º.

Thus, 2.5 of those seasonal hours would be 35.75º. Adding that to the initial rising point of 196.57º we get a rising point 232.32º. This can be looked up in our newly derived table to determine that this point is in Scorpio, specifically 39.63% of the way through which falls between the two bracketing latitudes we explored at the beginning of this paper affirming that our newly derived tables are correct.

Summary

The focus of this paper has had two parts. In the first part we explored the method Ptolemy described to calculate the point on the ecliptic rising at any time. However, it had the limitation that it was only usable when performed at particular latitudes as given in a series of tables. Therefore, in the second part, we explored the derivation of those tables, demonstrating how to use Menelaus' Theorem to get the first eight 10º arcs from the vernal equinox. To complete the first quadrant, we then employed a method using the length of the solstices. We then used symmetries to complete the rising time table.

While this exercise may have seemed rather narrow in scope, it serves as a whirlwind tour of some of the more important points of the *Almagest*'s first two books. To perform these calculations, we made use of Menelaus' Theorem (I.13), a table of arcs between the celestial equator and ecliptic (I.14 – I.15), a list of rising times at *sphaera recta* (I.16), and a collection of day lengths at varying latitudes (II.6). Although we have used a shortcut to prevent having to use more tables, we dealt with the *Crd arcs* from book I (I.10 – I.11). Most of these topics are some of the most fundamental to other problems and derivations has conducted throughout the first two books and serve as a strong foundation to any reader wanting to approach the text.

Appendix A - Mathematical Concepts & Techniques

Throughout the Almagest, Ptolemy makes use of several mathematical techniques which will be unfamiliar to most readers as they are either outdated or outside the standard mathematical curriculum. As such, I will introduce them here.

Sexagesimal Notation

Since the *Almagest* is a work about the movements on the celestial sphere, Ptolemy adopts a system of notation that is suited for spherical geometry. This system is known as sexagesimal and is base 60. While this does not seem intuitive, modern time is measured the same way, wherein 60 minutes = 1 hour. However, when dealing with a circle, there are 360º in a full rotation instead of 24h.

Thus, Ptolemy makes use of this system for all angle measurements wherein the first division (analogous to minutes) comes after a semi-colon and the second division (analogous to seconds and where the term originates) after a comma. For example, the value Ptolemy uses for the obliquity of the ecliptic (the angle between the celestial equator and ecliptic) is 23;51,20º.

While sexagesimal is not overly difficult for some mathematical functions such as addition and subtraction, it becomes cumbersome when doing multiplication and division. As such, it is often useful to convert this to modern, base 10 decimal. By and large in this paper I have stuck to decimals, but as there are some values that will need to be looked up in tables given in sexagesimal, conversions will be necessary.

To convert, the first division is divided by 60^1 and the second by 60^2. Using the example of the obliquity of the ecliptic above, the value in decimal notation would be $23 + 50/60 + 20/3600 =$ 23.86º.

The reverse conversion can be performed by taking off the whole number and multiplying the remaining decimal by 60. Then stripping off that whole number as the first division and repeating. Again, using the above example: 23; (.86 * 60) = 23;51.33 = 23;51, (.33 * 60) = 23;51,20º.

Spherical Geometry

One of the subjects that is not taught in a general mathematics curriculum is that of spherical geometry. This is a large topic, but a there are a few points that bear mention here. First is that the geometry most readers will have learned is meant for Cartesian planes, which is to say, flat. Since the surface of spheres is not flat, familiar rules will not apply. This means things like the sum of angles in a triangle adding up to more than 180º and being unable to use trigonometric functions. However, in spherical geometry, planes may be cut from the circle which are Cartesian.

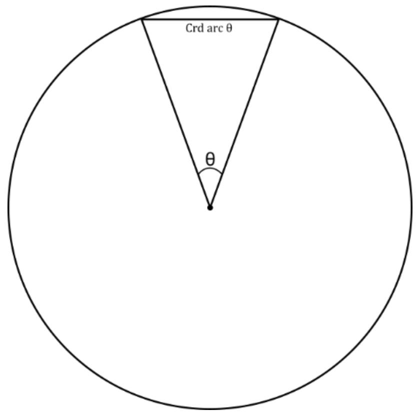
There are also a few terms which may prove useful. A **great circle** is a circle around the surface of a sphere whose center is coincident with the center of the sphere. For example, the equator is a great circle, but any other line of latitude is not. Any circle that is not a great circle is a **small circle**.

Ptolemaic Circles

Because Ptolemy frequently makes use of arcs along the surface of a sphere, care is taken as to how various features are defined. In particular, Ptolemy defines the circumference of a circle to be 360º. This is advantageous because it means that an arc of any great circle is the same length as the angle that subtends it. In addition, the radius is taken to be 60 parts. This seems like an odd number until we recall that Ptolemy works in sexagesimal, so 60 parts is akin to 1, making this similar to a unit circle.

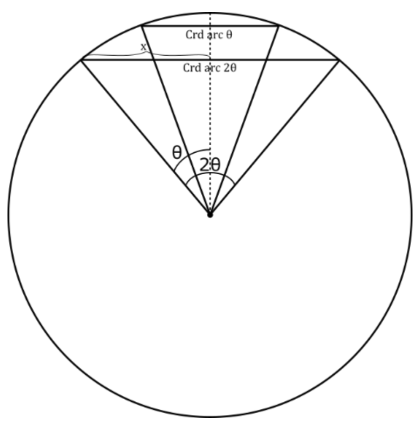
Chord Arc

Although trigonometry predates Ptolemy, it did not see widespread use until several centuries after his time. Thus, the *Almagest* is devoid of sin, cos, and tan functions despite how useful they may be. Instead, Ptolemy makes use of a related value known as a "chord arc" (abbreviated "Crd arc"). This is the length of a chord in a circle whose ends are defined by an arc which is subtended by a given angle.



In this paper, we will be making extensive use of these via Menelaus' Theorem (see below). Ptolemy finds the length of a chord from an arc or central angle (and vice versa) via a table of chords[19] (I.11) which he derived in I.10. However, while this table is broken up into half degree intervals with sixtieths listed to achieve more precision, it is a cumbersome methodology given that the value of the Crd arc can be easily obtained using trigonometry.

Particularly, Menelaus' Theorem requires the Crd arc of twice the angle, which is to say Crd arc 2θ. Fortunately we can quickly derive a formula to find this.



Here we first double the original angle to produce angle 2θ which subtends Crd arc 2θ. The resulting triangle is bisected to form a right triangle. Recalling that Ptolemy's circles are defined as having a radius of 60, we can then state that

$$\sin\theta = \frac{x}{60}$$

Solving for x:

$$x = 60\sin\theta$$

However, $Crd\ arc\ 2\theta = 2x$. Thus,

$$Crd\ arc\ 2\theta = 120\sin\theta$$

[19] See: http://jonvoisey.net/blog/2018/06/almagest-book-i-ptolemys-table-of-chords/

Conversely, the arc can be determined from the *Crd arc* by reversing this process. In other words:

$$\theta = sin^{-1}\left(\frac{Crd\ arc\ 2\theta}{120}\right)$$

Hours & Time Reckoning

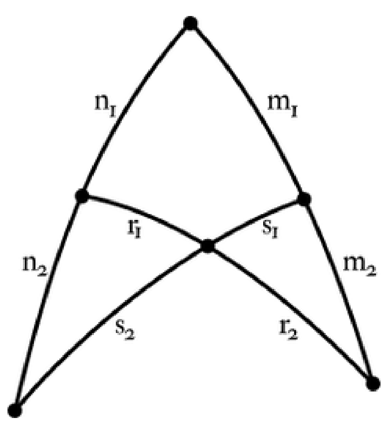
Ptolemy makes use of two systems of measuring the length of the day. The first is the system we take for granted in which a day[20] is divided into 24 hours of equal length in which case each hour is known as an equinoctial or equal hour.

Use is also made of another type of hour known as the seasonal or unequal hours in which the night and day are each divided into 12 hours. While this still leads to 24 hours in the day total, because of the longer days in summer this would mean a summer daytime hour is longer while a summer nighttime hour is shorter. The opposite would be true in winter.

Additionally, Ptolemy often tracks time in another system referred to as equinoctial time-degrees. What this system is truly tracking is the amount of the celestial equator that will rise in the amount of time it will take another section of a great circle. It is tracked in degrees and can represent time as the proportion of 360º that arc is, is the same proportion that time would be to 24 hours. Therefore, in this system the 360º in a full circle is akin to 24h in a day as the celestial sphere seem to make a full circle in that period of time. It then follows that each 15º is akin to 1 equinoctial hour.

Menelaus' Theorem

Menelaus' Theorem is one that Ptolemy makes extensive use of, which relates the intersection of arcs of four great circles in the configuration shown at right. It is derived in I.13[21] and has two versions. The first is given by the following equation:



$$\frac{Crd\ arc\ 2m}{Crd\ arc\ 2m_1} = \frac{Crd\ arc\ 2r}{Crd\ arc\ 2r_1} \times \frac{Crd\ arc\ 2s_2}{Crd\ arc\ 2s}$$

The second version is:

$$\frac{Crd\ arc\ 2m_2}{Crd\ arc\ 2m_1} = \frac{Crd\ arc\ 2r_2}{Crd\ arc\ 2r_1} \times \frac{Crd\ arc\ 2n_2}{Crd\ arc\ (n_2 + n_1)}$$

[20] Specifically, a solar day which is subsequent transits of the meridian by the sun, as opposed to a sidereal day which is subsequent transits of the meridian by distant stars.

[21] For derivations, see: http://jonvoisey.net/blog/2018/06/almagest-book-i-menelaus-theorem/

Appendix B – Daily Solar Position Table

This table gives the position of the sun along the ecliptic for dates starting from the vernal equinox (March 21). It also notes the zodiacal sign with each sign being defined as 30º, followed by the number of degrees past the beginning of the sign, as well as the percentage into the sign which is useful when using the rising times table.

It should be noted that this table is not accurate for modern day as the sun is no longer at the border of Pisces and Aries on the equinox due to the precession of the equinoxes which now places the vernal equinox in Pisces.

In addition, this table assumes that the sun moves evenly along the ecliptic. This is also incorrect as it moves more quickly when the Earth is closer to the sun. However, because the Earth's orbit is very nearly circular, the deviation from a mean motion is negligible.

Date	Days Past Equinox	Degrees	Sign	Degrees Into Sign	Percent Into Sign
3/21	0	0.00	Aries	0.00	0.00%
3/22	1	0.99	Aries	0.99	3.29%
3/23	2	1.97	Aries	1.97	6.58%
3/24	3	2.96	Aries	2.96	9.86%
3/25	4	3.95	Aries	3.95	13.15%
3/26	5	4.93	Aries	4.93	16.44%
3/27	6	5.92	Aries	5.92	19.73%
3/28	7	6.90	Aries	6.90	23.01%
3/29	8	7.89	Aries	7.89	26.30%
3/30	9	8.88	Aries	8.88	29.59%
3/31	10	9.86	Aries	9.86	32.88%
4/1	11	10.85	Aries	10.85	36.16%
4/2	12	11.84	Aries	11.84	39.45%
4/3	13	12.82	Aries	12.82	42.74%
4/4	14	13.81	Aries	13.81	46.03%
4/5	15	14.79	Aries	14.79	49.32%
4/6	16	15.78	Aries	15.78	52.60%
4/7	17	16.77	Aries	16.77	55.89%
4/8	18	17.75	Aries	17.75	59.18%
4/9	19	18.74	Aries	18.74	62.47%
4/10	20	19.73	Aries	19.73	65.75%
4/11	21	20.71	Aries	20.71	69.04%
4/12	22	21.70	Aries	21.70	72.33%
4/13	23	22.68	Aries	22.68	75.62%
4/14	24	23.67	Aries	23.67	78.90%
4/15	25	24.66	Aries	24.66	82.19%
4/16	26	25.64	Aries	25.64	85.48%
4/17	27	26.63	Aries	26.63	88.77%
4/18	28	27.62	Aries	27.62	92.05%
4/19	29	28.60	Aries	28.60	95.34%

Date	Days Past Equinox	Degrees	Sign	Degrees Into Sign	Percent Into Sign
4/20	30	29.59	Aries	29.59	98.63%
4/21	31	30.58	Taurus	0.58	1.92%
4/22	32	31.56	Taurus	1.56	5.21%
4/23	33	32.55	Taurus	2.55	8.49%
4/24	34	33.53	Taurus	3.53	11.78%
4/25	35	34.52	Taurus	4.52	15.07%
4/26	36	35.51	Taurus	5.51	18.36%
4/27	37	36.49	Taurus	6.49	21.64%
4/28	38	37.48	Taurus	7.48	24.93%
4/29	39	38.47	Taurus	8.47	28.22%
4/30	40	39.45	Taurus	9.45	31.51%
5/1	41	40.44	Taurus	10.44	34.79%
5/2	42	41.42	Taurus	11.42	38.08%
5/3	43	42.41	Taurus	12.41	41.37%
5/4	44	43.40	Taurus	13.40	44.66%
5/5	45	44.38	Taurus	14.38	47.95%
5/6	46	45.37	Taurus	15.37	51.23%
5/7	47	46.36	Taurus	16.36	54.52%
5/8	48	47.34	Taurus	17.34	57.81%
5/9	49	48.33	Taurus	18.33	61.10%
5/10	50	49.32	Taurus	19.32	64.38%
5/11	51	50.30	Taurus	20.30	67.67%
5/12	52	51.29	Taurus	21.29	70.96%
5/13	53	52.27	Taurus	22.27	74.25%
5/14	54	53.26	Taurus	23.26	77.53%
5/15	55	54.25	Taurus	24.25	80.82%
5/16	56	55.23	Taurus	25.23	84.11%
5/17	57	56.22	Taurus	26.22	87.40%
5/18	58	57.21	Taurus	27.21	90.68%
5/19	59	58.19	Taurus	28.19	93.97%

Date	Days Past Equinox	Degrees	Sign	Degrees Into Sign	Percent Into Sign
5/20	60	59.18	Taurus	29.18	97.26%
5/21	61	60.16	Gemini	0.16	0.55%
5/22	62	61.15	Gemini	1.15	3.84%
5/23	63	62.14	Gemini	2.14	7.12%
5/24	64	63.12	Gemini	3.12	10.41%
5/25	65	64.11	Gemini	4.11	13.70%
5/26	66	65.10	Gemini	5.10	16.99%
5/27	67	66.08	Gemini	6.08	20.27%
5/28	68	67.07	Gemini	7.07	23.56%
5/29	69	68.05	Gemini	8.05	26.85%
5/30	70	69.04	Gemini	9.04	30.14%
5/31	71	70.03	Gemini	10.03	33.42%
6/1	72	71.01	Gemini	11.01	36.71%
6/2	73	72.00	Gemini	12.00	40.00%
6/3	74	72.99	Gemini	12.99	43.29%
6/4	75	73.97	Gemini	13.97	46.58%
6/5	76	74.96	Gemini	14.96	49.86%
6/6	77	75.95	Gemini	15.95	53.15%
6/7	78	76.93	Gemini	16.93	56.44%
6/8	79	77.92	Gemini	17.92	59.73%
6/9	80	78.90	Gemini	18.90	63.01%
6/10	81	79.89	Gemini	19.89	66.30%
6/11	82	80.88	Gemini	20.88	69.59%
6/12	83	81.86	Gemini	21.86	72.88%
6/13	84	82.85	Gemini	22.85	76.16%
6/14	85	83.84	Gemini	23.84	79.45%
6/15	86	84.82	Gemini	24.82	82.74%
6/16	87	85.81	Gemini	25.81	86.03%
6/17	88	86.79	Gemini	26.79	89.32%
6/18	89	87.78	Gemini	27.78	92.60%
6/19	90	88.77	Gemini	28.77	95.89%
6/20	91	89.75	Gemini	29.75	99.18%
6/21	92	90.74	Cancer	0.74	2.47%
6/22	93	91.73	Cancer	1.73	5.75%
6/23	94	92.71	Cancer	2.71	9.04%
6/24	95	93.70	Cancer	3.70	12.33%
6/25	96	94.68	Cancer	4.68	15.62%
6/26	97	95.67	Cancer	5.67	18.90%
6/27	98	96.66	Cancer	6.66	22.19%
6/28	99	97.64	Cancer	7.64	25.48%
6/29	100	98.63	Cancer	8.63	28.77%
6/30	101	99.62	Cancer	9.62	32.05%
7/1	102	100.60	Cancer	10.60	35.34%
7/2	103	101.59	Cancer	11.59	38.63%
7/3	104	102.58	Cancer	12.58	41.92%
7/4	105	103.56	Cancer	13.56	45.21%
7/5	106	104.55	Cancer	14.55	48.49%
7/6	107	105.53	Cancer	15.53	51.78%
7/7	108	106.52	Cancer	16.52	55.07%
7/8	109	107.51	Cancer	17.51	58.36%
7/9	110	108.49	Cancer	18.49	61.64%
7/10	111	109.48	Cancer	19.48	64.93%
7/11	112	110.47	Cancer	20.47	68.22%
7/12	113	111.45	Cancer	21.45	71.51%
7/13	114	112.44	Cancer	22.44	74.79%
7/14	115	113.42	Cancer	23.42	78.08%
7/15	116	114.41	Cancer	24.41	81.37%
7/16	117	115.40	Cancer	25.40	84.66%
7/17	118	116.38	Cancer	26.38	87.95%
7/18	119	117.37	Cancer	27.37	91.23%
7/19	120	118.36	Cancer	28.36	94.52%
7/20	121	119.34	Cancer	29.34	97.81%
7/21	122	120.33	Leo	0.33	1.10%
7/22	123	121.32	Leo	1.32	4.38%
7/23	124	122.30	Leo	2.30	7.67%
7/24	125	123.29	Leo	3.29	10.96%
7/25	126	124.27	Leo	4.27	14.25%
7/26	127	125.26	Leo	5.26	17.53%
7/27	128	126.25	Leo	6.25	20.82%
7/28	129	127.23	Leo	7.23	24.11%
7/29	130	128.22	Leo	8.22	27.40%
7/30	131	129.21	Leo	9.21	30.68%
7/31	132	130.19	Leo	10.19	33.97%
8/1	133	131.18	Leo	11.18	37.26%
8/2	134	132.16	Leo	12.16	40.55%
8/3	135	133.15	Leo	13.15	43.84%
8/4	136	134.14	Leo	14.14	47.12%
8/5	137	135.12	Leo	15.12	50.41%
8/6	138	136.11	Leo	16.11	53.70%
8/7	139	137.10	Leo	17.10	56.99%
8/8	140	138.08	Leo	18.08	60.27%
8/9	141	139.07	Leo	19.07	63.56%
8/10	142	140.05	Leo	20.05	66.85%
8/11	143	141.04	Leo	21.04	70.14%

Date	Days Past Equinox	Degrees	Sign	Degrees Into Sign	Percent Into Sign
8/12	144	142.03	Leo	22.03	73.42%
8/13	145	143.01	Leo	23.01	76.71%
8/14	146	144.00	Leo	24.00	80.00%
8/15	147	144.99	Leo	24.99	83.29%
8/16	148	145.97	Leo	25.97	86.58%
8/17	149	146.96	Leo	26.96	89.86%
8/18	150	147.95	Leo	27.95	93.15%
8/19	151	148.93	Leo	28.93	96.44%
8/20	152	149.92	Leo	29.92	99.73%
8/21	153	150.90	Virgo	0.90	3.01%
8/22	154	151.89	Virgo	1.89	6.30%
8/23	155	152.88	Virgo	2.88	9.59%
8/24	156	153.86	Virgo	3.86	12.88%
8/25	157	154.85	Virgo	4.85	16.16%
8/26	158	155.84	Virgo	5.84	19.45%
8/27	159	156.82	Virgo	6.82	22.74%
8/28	160	157.81	Virgo	7.81	26.03%
8/29	161	158.79	Virgo	8.79	29.32%
8/30	162	159.78	Virgo	9.78	32.60%
8/31	163	160.77	Virgo	10.77	35.89%
9/1	164	161.75	Virgo	11.75	39.18%
9/2	165	162.74	Virgo	12.74	42.47%
9/3	166	163.73	Virgo	13.73	45.75%
9/4	167	164.71	Virgo	14.71	49.04%
9/5	168	165.70	Virgo	15.70	52.33%
9/6	169	166.68	Virgo	16.68	55.62%
9/7	170	167.67	Virgo	17.67	58.90%
9/8	171	168.66	Virgo	18.66	62.19%
9/9	172	169.64	Virgo	19.64	65.48%
9/10	173	170.63	Virgo	20.63	68.77%
9/11	174	171.62	Virgo	21.62	72.05%
9/12	175	172.60	Virgo	22.60	75.34%
9/13	176	173.59	Virgo	23.59	78.63%
9/14	177	174.58	Virgo	24.58	81.92%
9/15	178	175.56	Virgo	25.56	85.21%
9/16	179	176.55	Virgo	26.55	88.49%
9/17	180	177.53	Virgo	27.53	91.78%
9/18	181	178.52	Virgo	28.52	95.07%
9/19	182	179.51	Virgo	29.51	98.36%
9/20	183	180.49	Libra	0.49	1.64%
9/21	184	181.48	Libra	1.48	4.93%
9/22	185	182.47	Libra	2.47	8.22%

Date	Days Past Equinox	Degrees	Sign	Degrees Into Sign	Percent Into Sign
9/23	186	183.45	Libra	3.45	11.51%
9/24	187	184.44	Libra	4.44	14.79%
9/25	188	185.42	Libra	5.42	18.08%
9/26	189	186.41	Libra	6.41	21.37%
9/27	190	187.40	Libra	7.40	24.66%
9/28	191	188.38	Libra	8.38	27.95%
9/29	192	189.37	Libra	9.37	31.23%
9/30	193	190.36	Libra	10.36	34.52%
10/1	194	191.34	Libra	11.34	37.81%
10/2	195	192.33	Libra	12.33	41.10%
10/3	196	193.32	Libra	13.32	44.38%
10/4	197	194.30	Libra	14.30	47.67%
10/5	198	195.29	Libra	15.29	50.96%
10/6	199	196.27	Libra	16.27	54.25%
10/7	200	197.26	Libra	17.26	57.53%
10/8	201	198.25	Libra	18.25	60.82%
10/9	202	199.23	Libra	19.23	64.11%
10/10	203	200.22	Libra	20.22	67.40%
10/11	204	201.21	Libra	21.21	70.68%
10/12	205	202.19	Libra	22.19	73.97%
10/13	206	203.18	Libra	23.18	77.26%
10/14	207	204.16	Libra	24.16	80.55%
10/15	208	205.15	Libra	25.15	83.84%
10/16	209	206.14	Libra	26.14	87.12%
10/17	210	207.12	Libra	27.12	90.41%
10/18	211	208.11	Libra	28.11	93.70%
10/19	212	209.10	Libra	29.10	96.99%
10/20	213	210.08	Scorpio	0.08	0.27%
10/21	214	211.07	Scorpio	1.07	3.56%
10/22	215	212.05	Scorpio	2.05	6.85%
10/23	216	213.04	Scorpio	3.04	10.14%
10/24	217	214.03	Scorpio	4.03	13.42%
10/25	218	215.01	Scorpio	5.01	16.71%
10/26	219	216.00	Scorpio	6.00	20.00%
10/27	220	216.99	Scorpio	6.99	23.29%
10/28	221	217.97	Scorpio	7.97	26.58%
10/29	222	218.96	Scorpio	8.96	29.86%
10/30	223	219.95	Scorpio	9.95	33.15%
10/31	224	220.93	Scorpio	10.93	36.44%
11/1	225	221.92	Scorpio	11.92	39.73%
11/2	226	222.90	Scorpio	12.90	43.01%
11/3	227	223.89	Scorpio	13.89	46.30%

Date	Days Past Equinox	Degrees	Sign	Degrees Into Sign	Percent Into Sign
11/4	228	224.88	Scorpio	14.88	49.59%
11/5	229	225.86	Scorpio	15.86	52.88%
11/6	230	226.85	Scorpio	16.85	56.16%
11/7	231	227.84	Scorpio	17.84	59.45%
11/8	232	228.82	Scorpio	18.82	62.74%
11/9	233	229.81	Scorpio	19.81	66.03%
11/10	234	230.79	Scorpio	20.79	69.32%
11/11	235	231.78	Scorpio	21.78	72.60%
11/12	236	232.77	Scorpio	22.77	75.89%
11/13	237	233.75	Scorpio	23.75	79.18%
11/14	238	234.74	Scorpio	24.74	82.47%
11/15	239	235.73	Scorpio	25.73	85.75%
11/16	240	236.71	Scorpio	26.71	89.04%
11/17	241	237.70	Scorpio	27.70	92.33%
11/18	242	238.68	Scorpio	28.68	95.62%
11/19	243	239.67	Scorpio	29.67	98.90%
11/20	244	240.66	Sagittarius	0.66	2.19%
11/21	245	241.64	Sagittarius	1.64	5.48%
11/22	246	242.63	Sagittarius	2.63	8.77%
11/23	247	243.62	Sagittarius	3.62	12.05%
11/24	248	244.60	Sagittarius	4.60	15.34%
11/25	249	245.59	Sagittarius	5.59	18.63%
11/26	250	246.58	Sagittarius	6.58	21.92%
11/27	251	247.56	Sagittarius	7.56	25.21%
11/28	252	248.55	Sagittarius	8.55	28.49%
11/29	253	249.53	Sagittarius	9.53	31.78%
11/30	254	250.52	Sagittarius	10.52	35.07%
12/1	255	251.51	Sagittarius	11.51	38.36%
12/2	256	252.49	Sagittarius	12.49	41.64%
12/3	257	253.48	Sagittarius	13.48	44.93%
12/4	258	254.47	Sagittarius	14.47	48.22%
12/5	259	255.45	Sagittarius	15.45	51.51%
12/6	260	256.44	Sagittarius	16.44	54.79%
12/7	261	257.42	Sagittarius	17.42	58.08%
12/8	262	258.41	Sagittarius	18.41	61.37%
12/9	263	259.40	Sagittarius	19.40	64.66%
12/10	264	260.38	Sagittarius	20.38	67.95%
12/11	265	261.37	Sagittarius	21.37	71.23%
12/12	266	262.36	Sagittarius	22.36	74.52%
12/13	267	263.34	Sagittarius	23.34	77.81%
12/14	268	264.33	Sagittarius	24.33	81.10%
12/15	269	265.32	Sagittarius	25.32	84.38%

Date	Days Past Equinox	Degrees	Sign	Degrees Into Sign	Percent Into Sign
12/16	270	266.30	Sagittarius	26.30	87.67%
12/17	271	267.29	Sagittarius	27.29	90.96%
12/18	272	268.27	Sagittarius	28.27	94.25%
12/19	273	269.26	Sagittarius	29.26	97.53%
12/20	274	270.25	Capricorn	0.25	0.82%
12/21	275	271.23	Capricorn	1.23	4.11%
12/22	276	272.22	Capricorn	2.22	7.40%
12/23	277	273.21	Capricorn	3.21	10.68%
12/24	278	274.19	Capricorn	4.19	13.97%
12/25	279	275.18	Capricorn	5.18	17.26%
12/26	280	276.16	Capricorn	6.16	20.55%
12/27	281	277.15	Capricorn	7.15	23.84%
12/28	282	278.14	Capricorn	8.14	27.12%
12/29	283	279.12	Capricorn	9.12	30.41%
12/30	284	280.11	Capricorn	10.11	33.70%
12/31	285	281.10	Capricorn	11.10	36.99%
1/1	286	282.08	Capricorn	12.08	40.27%
1/2	287	283.07	Capricorn	13.07	43.56%
1/3	288	284.05	Capricorn	14.05	46.85%
1/4	289	285.04	Capricorn	15.04	50.14%
1/5	290	286.03	Capricorn	16.03	53.42%
1/6	291	287.01	Capricorn	17.01	56.71%
1/7	292	288.00	Capricorn	18.00	60.00%
1/8	293	288.99	Capricorn	18.99	63.29%
1/9	294	289.97	Capricorn	19.97	66.58%
1/10	295	290.96	Capricorn	20.96	69.86%
1/11	296	291.95	Capricorn	21.95	73.15%
1/12	297	292.93	Capricorn	22.93	76.44%
1/13	298	293.92	Capricorn	23.92	79.73%
1/14	299	294.90	Capricorn	24.90	83.01%
1/15	300	295.89	Capricorn	25.89	86.30%
1/16	301	296.88	Capricorn	26.88	89.59%
1/17	302	297.86	Capricorn	27.86	92.88%
1/18	303	298.85	Capricorn	28.85	96.16%
1/19	304	299.84	Capricorn	29.84	99.45%
1/20	305	300.82	Aquarius	0.82	2.74%
1/21	306	301.81	Aquarius	1.81	6.03%
1/22	307	302.79	Aquarius	2.79	9.32%
1/23	308	303.78	Aquarius	3.78	12.60%
1/24	309	304.77	Aquarius	4.77	15.89%
1/25	310	305.75	Aquarius	5.75	19.18%
1/26	311	306.74	Aquarius	6.74	22.47%

Date	Days Past Equinox	Degrees	Sign	Degrees Into Sign	Percent Into Sign
1/27	312	307.73	Aquarius	7.73	25.75%
1/28	313	308.71	Aquarius	8.71	29.04%
1/29	314	309.70	Aquarius	9.70	32.33%
1/30	315	310.68	Aquarius	10.68	35.62%
1/31	316	311.67	Aquarius	11.67	38.90%
2/1	317	312.66	Aquarius	12.66	42.19%
2/2	318	313.64	Aquarius	13.64	45.48%
2/3	319	314.63	Aquarius	14.63	48.77%
2/4	320	315.62	Aquarius	15.62	52.05%
2/5	321	316.60	Aquarius	16.60	55.34%
2/6	322	317.59	Aquarius	17.59	58.63%
2/7	323	318.58	Aquarius	18.58	61.92%
2/8	324	319.56	Aquarius	19.56	65.21%
2/9	325	320.55	Aquarius	20.55	68.49%
2/10	326	321.53	Aquarius	21.53	71.78%
2/11	327	322.52	Aquarius	22.52	75.07%
2/12	328	323.51	Aquarius	23.51	78.36%
2/13	329	324.49	Aquarius	24.49	81.64%
2/14	330	325.48	Aquarius	25.48	84.93%
2/15	331	326.47	Aquarius	26.47	88.22%
2/16	332	327.45	Aquarius	27.45	91.51%
2/17	333	328.44	Aquarius	28.44	94.79%
2/18	334	329.42	Aquarius	29.42	98.08%
2/19	335	330.41	Pisces	0.41	1.37%
2/20	336	331.40	Pisces	1.40	4.66%
2/21	337	332.38	Pisces	2.38	7.95%
2/22	338	333.37	Pisces	3.37	11.23%

Date	Days Past Equinox	Degrees	Sign	Degrees Into Sign	Percent Into Sign
2/23	339	334.36	Pisces	4.36	14.52%
2/24	340	335.34	Pisces	5.34	17.81%
2/25	341	336.33	Pisces	6.33	21.10%
2/26	342	337.32	Pisces	7.32	24.38%
2/27	343	338.30	Pisces	8.30	27.67%
2/28	344	339.29	Pisces	9.29	30.96%
3/1	345	340.27	Pisces	10.27	34.25%
3/2	346	341.26	Pisces	11.26	37.53%
3/3	347	342.25	Pisces	12.25	40.82%
3/4	348	343.23	Pisces	13.23	44.11%
3/5	349	344.22	Pisces	14.22	47.40%
3/6	350	345.21	Pisces	15.21	50.68%
3/7	351	346.19	Pisces	16.19	53.97%
3/8	352	347.18	Pisces	17.18	57.26%
3/9	353	348.16	Pisces	18.16	60.55%
3/10	354	349.15	Pisces	19.15	63.84%
3/11	355	350.14	Pisces	20.14	67.12%
3/12	356	351.12	Pisces	21.12	70.41%
3/13	357	352.11	Pisces	22.11	73.70%
3/14	358	353.10	Pisces	23.10	76.99%
3/15	359	354.08	Pisces	24.08	80.27%
3/16	360	355.07	Pisces	25.07	83.56%
3/17	361	356.05	Pisces	26.05	86.85%
3/18	362	357.04	Pisces	27.04	90.14%
3/19	363	358.03	Pisces	28.03	93.42%
3/20	364	359.01	Pisces	29.01	96.71%

Appendix C – Rising Time Tables

The following tables are transcribed from II.8 of the *Almagest*. The locations are listed along the top with the length of the longest day and the latitude beneath. Along the vertical axis we have each sign, broken up into 10º intervals.

In the main body of the table, for each location, we first list the amount of time it takes each 10º arc to rise, as well as the cumulative time (in time-degrees) it would take to rise to that point beginning at the vernal equinox.

As with the previous table, this one is also inaccurate for modern use as it too depicts the vernal equinox at the beginning of Aries.

Table begins on next page to preserve formatting.

Regiomontanus, Epitome of the Almagest c1496

Sign	10º	*Sphaera Recta* 12h º	0º Acc	Avalite Gulf 12.5h º	8;25º Acc	Meroe 13h º	16;27º Acc
Aries	10º	9;10	9;10	8;35	8;35	7;58	7;58
	20º	9;15	18;25	8;39	17;14	8;5	16;3
	30º	9;25	27;50	8;52	26;6	8;17	24;20
Taurus	10º	9;40	37;30	9;8	35;14	8;36	32;56
	20º	9;58	47;28	9;29	44;43	9;1	41;57
	30º	10;16	57;44	9;51	54;34	9;27	51;24
Gemini	10º	10;34	68;18	10;15	64;49	9;56	61;20
	20º	10;47	79;5	10;35	75;24	10;23	71;43
	30º	10;55	90;0	10;51	86;15	10;47	82;30
Cancer	10º	10;55	100;55	10;59	97;14	11;3	93;33
	20º	10;47	111;42	10;59	108;13	11;11	104;44
	30º	10;34	122;16	10;53	119;6	11;12	115;56
Leo	10º	10;16	132;32	10;41	129;47	11;5	127;1
	20º	9;58	142;30	10;27	140;14	10;55	137;56
	30º	9;40	125;10	10;12	150;26	10;44	148;40
Virgo	10º	9;25	161;35	9;58	160;24	10;33	159;13
	20º	9;15	170;50	9;51	170;15	10;25	169;38
	30º	9;10	180;0	9;45	180;0	10;22	180;0
Libra	10º	9;10	189;10	9;45	189;45	10;22	190;22
	20º	9;15	198;25	9;51	199;36	10;25	200;47
	30º	9;25	207;50	9;58	209;34	10;33	211;20
Scorpius	10º	9;40	217;30	10;12	219;46	10;44	222;4
	20º	9;58	227;28	10;27	230;13	10;55	232;59
	30º	10;16	237;44	10;41	240;54	11;5	244;4
Sagittarius	10º	10;34	248;18	10;53	251;47	11;12	255;16
	20º	10;47	259;5	10;59	262;46	11;11	266;27
	30º	10;55	270;0	10;59	273;45	11;3	277;30
Capricornus	10º	10;55	280;55	10;51	284;36	10;47	288;17
	20º	10;47	291;42	10;35	295;11	10;23	298;40
	30º	10;34	302;16	10;15	305;26	9;56	308;36
Aquarius	10º	10;16	312;32	9;51	315;17	9;27	318;3
	20º	9;58	322;30	9;29	324;46	9;1	327;4
	30º	9;40	332;10	9;8	333;54	8;36	335;40
Pisces	10º	9;25	341;35	8;52	342;46	8;17	343;57
	20º	9;15	350;50;	8;39	351;25	8;5	352;2
	30º	9;10	360;0	8;35	360;0	7;58	360;0

Sign	10º	Soene 13.5h º	23;51º Acc	Lower Egypt 14h º	30;22º Acc	Rhodes 14.5h º	36;0º Acc
Aries	10º	7;23	7;23	6;48	6;48	6;14	6;14
	20º	7;29	14;52	6;55	13;43	6;21	12;35
	30º	7;45	22;37	7;10	20;53	6;37	19;12
Taurus	10º	8;4	30;41	7;33	28;26	7;1	26;13
	20º	8;31	39;12	8;2	36;28	7;33	33;46
	30º	9;3	48;15	8;37	45;5	8;12	41;58
Gemini	10º	9;36	57;51	9;17	54;22	8;56	50;54
	20º	10;11	68;2	10;0	64;22	9;47	60;41
	30º	10;43	78;45	10;38	75;0	10;34	71;15
Cancer	10º	11;7	89;52	11;12	86;12	11;16	82;31
	20º	11;23	101;15	11;34	97;46	11;47	94;18
	30º	11;32	112;47	11;51	109;37	12;12	106;30
Leo	10º	11;29	124;16	11;55	121;32	12;20	118;50
	20º	11;25	135;41	11;54	133;26	12;23	131;13
	30º	11;16	146;57	11;47	145;13	12;19	143;32
Virgo	10º	11;5	158;2	11;40	156;53	12;13	155;45
	20º	11;1	169;3	11;35	168;28	12;9	167;54
	30º	10;57	180;0	11;32	180;0	12;6	180;0
Libra	10º	10;57	190;57	11;32	191;32	12;6	192;6
	20º	11;1	201;58	11;35	203;7	12;9	204;15
	30º	11;5	213;3	11;40	214;47	12;13	216;28
Scorpius	10º	11;16	224;19	11;47	226;34	12;19	228;47
	20º	11;25	235;44	11;54	238;28	12;23	241;10
	30º	11;29	247;13	11;55	250;23	12;20	253;30
Sagittarius	10º	11;32	258;45	11;51	262;14	12;12	265;42
	20º	11;23	270;8	11;34	273;48	11;47	277;29
	30º	11;7	281;15	11;12	285;0	11;16	288;45
Capricornus	10º	10;43	291;58	10;38	295;38	10;34	299;19
	20º	10;11	302;9	10;0	305;38	9;47	309;6
	30º	9;36	311;45	9;17	314;55	8;56	318;2
Aquarius	10º	9;3	320;48	8;37	323;32	8;12	326;14
	20º	8;31	329;19	8;2	331;34	7;33	333;47
	30º	8;4	337;23	7;33	339;7	7;1	340;48
Pisces	10º	7;45	345;8	7;10	346;17	6;37	347;25
	20º	7;29	352;37	6;55	353;12	6;21	353;46
	30º	7;23	360;0	6;48	360;0	6;14	360;0

Sign	10º	Hellespont 15h º	40;56º Acc	Middle of Pontus 15.5h º	45;1º Acc	Middle of Borysthenes 16h º	48;32º Acc
Aries	10º	5;40	5;40	5;8	5;8	4;36	4;36
	20º	5;47	11;27	5;14	10;22	4;43	9;19
	30º	6;5	17;32	5;33	15;55	5;1	14;20
Taurus	10º	6;29	24;1	5;58	21;53	5;26	19;46
	20º	7;4	31;5	6;34	28;27	6;5	25;51
	30º	7;46	38;51	7;20	35;47	6;52	32;43
Gemini	10º	8;38	47;29	8;15	44;2	7;53	40;36
	20º	9;32	57;1	9;19	53;21	9;5	49;41
	30º	10;29	67;30	10;24	63;45	10;19	60;0
Cancer	10º	11;21	78;51	11;26	75;11	11;31	71;31
	20º	12;2	90;53	12;15	87;26	12;29	84;0
	30º	12;30	103;23	12;53	100;19	13;15	97;15
Leo	10º	12;46	116;9	13;12	113;31	13;40	110;55
	20º	12;52	129;1	13;22	126;53	13;51	124;46
	30º	12;51	141;52	13;22	140;15	13;54	138;40
Virgo	10º	12;45	154;37	13;17	153;32	13;49	152;29
	20º	12;43	167;20	13;16	166;48	13;47	166;16
	30º	12;40	180;0	13;12	180;0	13;44	180;0
Libra	10º	12;40	192;40	13;12	193;12	13;44	193;44
	20º	12;43	205;23	13;16	206;28	13;47	207;31
	30º	12;45	218;8	13;17	219;45	13;49	221;20
Scorpius	10º	12;51	230;59	13;22	233;7	13;54	235;14
	20º	12;52	243;51	13;22	246;29	13;51	249;5
	30º	12;46	256;37	13;12	259;41	13;40	262;45
Sagittarius	10º	12;30	269;7	12;53	272;34	13;15	276;0
	20º	12;2	281;9	12;15	284;49	12;29	288;29
	30º	11;21	292;30	11;26	296;15	11;31	300;0
Capricornus	10º	10;29	302;59	10;24	306;39	10;19	310;19
	20º	9;32	312;31	9;19	315;58	9;5	319;24
	30º	8;38	321;9	8;15	324;13	7;53	327;17
Aquarius	10º	7;46	328;55	7;20	331;33	6;52	334;9
	20º	7;4	335;59	6;34	338;7	6;5	340;14
	30º	6;29	342;28	5;58	344;5	5;26	345;40
Pisces	10º	6;5	348;33	5;33	349;38	5;1	350;41
	20º	5;47	354;20	5;14	354;52	4;43	355;24
	30º	5;40	360;0	5;8	360;0	4;36	360;0

Sign	10º	Southernmost Brittania 16.5h º	51;30º Acc	Mouths of Tanais 17h º	54;1º Acc
Aries	10º	4;5	4;5	3;36	3;36
	20º	4;12	8;17	3;43	7;19
	30º	4;31	12;48	4;0	11;19
Taurus	10º	4;56	17;44	4;26	15;45
	20º	5;34	23;18	5;4	20;49
	30º	6;25	29;43	5;56	26;45
Gemini	10º	7;29	37;12	7;5	33;50
	20º	8;49	46;1	8;33	42;23
	30º	10;14	56;15	10;7	52;30
Cancer	10º	11;36	67;51	11;43	64;13
	20º	12;45	80;36	13;1	77;14
	30º	13;39	94;15	14;3	91;17
Leo	10º	14;7	108;22	14;36	105;53
	20º	14;22	122;44	14;52	120;45
	30º	1424;	137;8	14;54	135;39
Virgo	10º	14;19	151;27	14;50	150;29
	20º	14;18	165;45	14;47	165;16
	30º	14;15	180;0	14;44	180;0
Libra	10º	14;15	194;15	14;44	194;44
	20º	14;18	208;33	14;47	209;31
	30º	14;19	222;52	14;50	224;21
Scorpius	10º	14;24	237;16	14;54	239;15
	20º	14;22	251;31	14;52	254;7
	30º	14;7	265;45	14;36	268;43
Sagittarius	10º	13;39	279;24	14;3	282;46
	20º	12;45	292;9	13;1	295;47
	30º	11;36	303;45	11;43	307;30
Capricornus	10º	10;14	313;59	10;7	317;37
	20º	8;49	322;48	8;33	326;10
	30º	7;29	330;17	7;5	333;15
Aquarius	10º	6;25	336;42	5;56	339;11
	20º	5;34	342;16	5;4	344;15
	30º	4;56	347;12	4;26	348;41
Pisces	10º	3;41	351;43	4;0	352;41
	20º	4;12	355;55	3;43	356;24
	30º	4;5	360;0	3;36	360;0

Appendix D – Table of Arc Lengths Between the Celestial Equator and Ecliptic

This is table I.15 which, given an arc of the ecliptic from the vernal equinox, allows one to look up the arc between the ecliptic and celestial equator.

Arc of the Ecliptic	Arc of the Meridian (Sexagesimal)	Arc of the Meridian (Degrees)
1	0;24,16	0.40
2	0;48,31	0.81
3	1;12,46	1.21
4	1;37,0	1.62
5	2;1,12	2.02
6	2;25,22	2.42
7	2;49,30	2.83
8	3;13,35	3.23
9	3;37,37	3.63
10	4;1,38	4.03
11	4;25,32	4.43
12	4;49,24	4.82
13	5;13,11	5.22
14	5;36,53	5.61
15	6;0,31	6.01
16	6;24,1	6.40
17	6;47,26	6.79
18	7;10,45	7.18
19	7;33,57	7.57
20	7;57,3	7.95
21	8;20,0	8.33
22	8;42,50	8.71
23	9;5,32	9.09
24	9;28,5	9.47
25	9;50,29	9.84
26	10;12,46	10.21
27	10;34,57	10.58
28	10;56,44	10.95
29	11;18,25	11.31
30	11;39,59	11.67
31	12;1,20	12.02
32	12;22,30	12.38
33	12;43,28	12.72
34	13;4,14	13.07
35	13;24,47	13.41
36	13;45,6	13.75

Arc of the Ecliptic	Arc of the Meridian (Sexagesimal)	Arc of the Meridian (Degrees)
37	14;5,11	14.09
28	14;25,2	14.42
39	14;44,39	14.74
40	15;4,4	15.07
41	15;23,10	15.39
42	15;42,2	15.70
43	16;0,38	16.01
44	16;18,58	16.32
45	16;37,1	16.62
46	16;54,47	16.91
47	17;12,16	17.20
48	17;29,27	17.49
49	17;46,20	17.77
50	18;2,53	18.05
51	18;19,15	18.32
52	18;35,5	18.58
53	18;50,41	18.84
54	19;5,57	19.10
55	19;20,56	19.35
56	19;35,28	19.59
57	19;49,42	19.83
58	20;3,31	20.06
59	20;17,4	20.28
60	20;30,9	20.50
61	20;42,58	20.72
62	20;55,24	20.92
63	21;7,21	21.12
64	21;18,58	21.32
65	21;30,11	21.50
66	21;41,0	21.68
67	21;51,25	21.86
68	22;1,25	22.02
69	22;11,1	22.18
70	22;20,11	22.34
71	22;28,57	22.48
72	22;37,17	22.62

Arc of the Ecliptic	Arc of the Meridian (Sexagesimal)	Arc of the Meridian (Degrees)
73	22;45,11	22.75
74	22;52,39	22.88
75	22;59,41	22.99
76	23;6,17	23.10
77	23;12,27	23.21
78	23;18,11	23.30
79	23;23,28	23.39
80	23;28,16	23.47
81	23;32,30	23.54
82	23;36,35	23.61
83	23;40,2	23.67
84	23;43,2	23.72
85	23;45,34	23.76
86	23;47,39	23.79
87	23;49,16	23.82
88	23;50,25	23.84
89	23;51,6	23.85
90	23;51,20	23.86

Appendix E – Rising Times at *Sphaera Recta*

This table is compiled from I.16.

Arc	Rising Time (Sexagesimal)	Rising Time (Decimal)	Cumulative (Sexagesimal)
10	9;10	9.17	9;10
20	9;15	9.25	18;25
30	9;25	9.42	27;50
40	9;40	9.67	37;30
50	9;58	9.97	47;28
60	10;16	10.27	57;44
70	10;34	10.57	68;18
80	10;47	10.78	79;5
90	10;55	10.92	90;0

Appendix F – Length of longest day at various latitudes

This table presents the values for the length of the longest days as described in II.6.

Latitude	LoLD
0.00	12.00
4.50	12.25
8.42	12.50
12.50	12.75
16.45	13.00
20.23	13.25
23.85	13.50
27.20	13.75
30.37	14.00
33.30	14.25
36.00	14.50
38.58	14.75
40.93	15.00
43.02	15.25
45.02	15.50
46.85	15.75
48.53	16.00
50.07	16.25
51.50	16.50
52.83	16.75
54.02	17.00
55.00	17.25
56.00	17.50
57.00	17.75
58.00	18.00
59.50	18.50
61.00	19.00
62.00	19.50
63.00	20.00
64.50	21.00
66.00	23.00
66.14	24.00